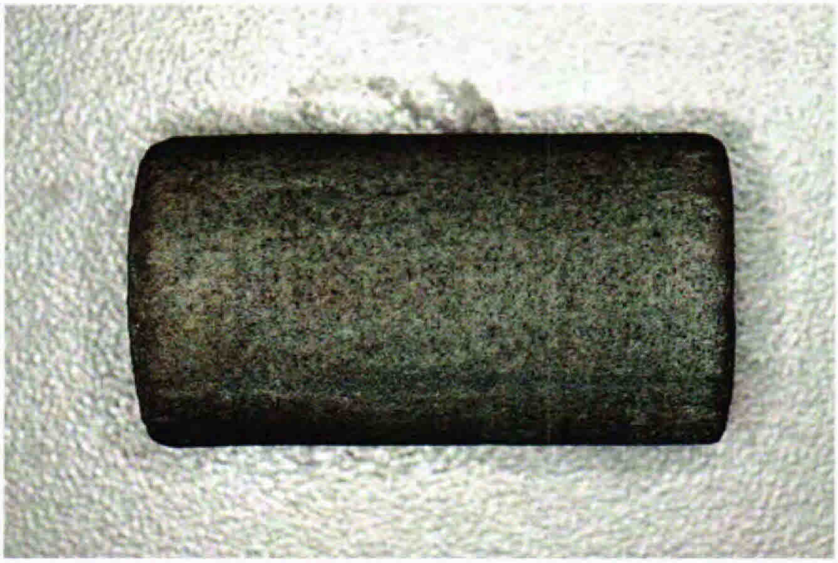
In re: Oil Spill by the Oil Rig "Deepwater Horizon" in the Gulf of Mexico, on April 20, 2010

UNITED STATES DISTRICT COURT
EASTERN DISTRICT OF LOUISIANA
MDL NO. 2179, SECTION J
JUDGE BARBIER; MAGISTRATE JUDGE SHUSHAN

Pore Volume Compressibility of the Macondo Reservoir

Expert Report of
Robert W. Zimmerman
Department of Earth Sciences and Engineering
Imperial College of Science, Technology and Medicine
London SW7 2AZ, UK

May 1, 2013



CONFIDENTIAL

11497
Exhibit No.
Worldwide Court Reporters, Inc.

Table of Contents

1. Introduction and Statement of Purpose

I have been retained on behalf of BP Exploration & Production to evaluate laboratory data and offer opinions about the value of a rock property called "uniaxial pore volume compressibility" or "UPVC" (sometimes referred to as "rock compressibility" or "formation compressibility") for the Macondo reservoir sandstone. The value of UPVC is relevant to various reservoir engineering calculations, including material balance calculations. Government expert witnesses have in several cases used a number for UPVC that is approximately double the value that is indicated by measurements on actual samples of Macondo reservoir rock. The Government expert reports do not provide an expert analysis as to why they depart from the measured data. In this report I analyze the data from three tests performed by an independent laboratory that allow a calculation of the UPVC. All of these strands of analysis converge on the number that I have provided to other BP experts as an input into their calculations.

2. Personal Background – Robert W. Zimmerman

I am currently Professor of Rock Mechanics in the Department of Earth Science and Engineering, Royal School of Mines, Imperial College, London, United Kingdom. Rock mechanics is the name of the field of study that encompasses the subject of pore volume compressibility, which is one of the properties of the Macondo reservoir that is in dispute between the calculations of cumulative flow by BP's experts and those of several United States experts.

I have a B.S. degree in Mechanical Engineering from Columbia University, an M.S. degree in Mechanical Engineering from Columbia University, and a Ph.D. in Solid Mechanics from the University of California at Berkeley. My Ph.D. thesis was entitled "The Effect of Pore Structure on the Pore and Bulk Compressibilities of Consolidated Sandstones".

I have previously been a lecturer at the University of California, Berkeley, a staff scientist at the Lawrence Berkeley National Laboratory, and Head of the Division of Engineering Geology and Geophysics at the Royal Institute of Technology (KTH) in Stockholm. I have taught courses in rock mechanics, rock physics, geodynamics, soil mechanics, engineering mechanics, fluid flow through porous media, heat transfer, fluid mechanics, and applied mathematics, at UC Berkeley, KTH, and Imperial College.

I currently conduct research in rock mechanics, fractured rock hydrology, and petrophysics, with applications to petroleum engineering, mining, nuclear waste disposal, and carbon sequestration. I have authored or co-authored ninety papers in refereed scientific journals, and eighty-five conference papers, including thirteen SPE papers. I have supervised or co-supervised twenty-six Ph.D. students (twenty-one completed, five in progress), mainly in various aspects of rock mechanics.

I have been, since 2006, the Editor-in-Chief of the *International Journal of Rock Mechanics and Mining Sciences*, and serve on the Editorial Boards of *Transport in Porous Media* and the *International Journal of Engineering Science*. I am the author of the monograph *Compressibility of Sandstones*, published as Volume 29 of the series Developments in Petroleum Science (Elsevier, 1991), and am the co-author, with J. C. Jaeger and N. G. W. Cook, of *Fundamentals of Rock Mechanics* (4th ed., Wiley-Blackwell, 2007).

My papers and books have received over 3000 citations, according to the Institute of Scientific Information, and over 4500 citations according to Google Scholar. In 2010 I was awarded the Maurice A. Biot Medal for Poromechanics from the American Society of Civil Engineers, in recognition of my "outstanding contributions in applying poroelasticity to rock mechanics and fluid flow in fractured media".

My detailed curriculum vitae is appended to this report as Appendix 2.

3. Executive Summary

Uniaxial pore volume compressibility (UPVC) is a key parameter in many petroleum reservoir engineering calculations, including material balance calculations and well test analysis. In the present context, UPVC represents how the pore volume of reservoir rock changes as the pore pressure inside the rock changes while the reservoir is depleting. UPVC is typically expressed in units of microsips, which are sometimes abbreviated "μsips". One microsip is equal to 1×10^{-6}/psi. The UPVC of the Macondo reservoir sandstone is a necessary data input for most reservoir analysis techniques for estimating the cumulative amount of oil that flowed from the Macondo well from April 20, 2010, to July 15, 2010.

Counsel for BP have asked me to analyze and evaluate laboratory and other data, including laboratory measurements of rock samples performed by Weatherford Laboratories, to determine the best estimate of the Macondo reservoir sandstone's average UPVC. The primary opinions I have formed are as follows:

i. It is common in petroleum engineering to test rotary sidewall core rock samples — the type of samples available from the Macondo Reservoir — for pore volume compressibility and to analyze and rely on the results of those tests.

ii. Based on measurements by Weatherford Laboratories of pre-incident rotary sidewall core rock samples, 6.35 microsips is the best estimate of the average UPVC of the Macondo Reservoir.

iii. My estimate of an average UPVC of 6.35 microsips is supported by analysis of three independent laboratory measurements. First, I have analyzed the raw data of the uniaxial compression tests conducted by Weatherford Laboratories on the pre-incident Macondo rock samples, and I have determined that the average UPVCs of these samples range from 4.34 microsips to 8.57 microsips, with an average value of 6.35 microsips.

iv. Second, I have analyzed Weatherford's measurements of the porosity changes that occurred in the Macondo rock samples under hydrostatic compression, and used well-established relationships (physical and mathematical) to convert those measurements to a UPVC value. I determined that the UPVC value derived from this set of data was slightly lower than 6.35 microsips.

v. And third, I have calculated the Macondo UPVC by analyzing Weatherford's measurements of ultrasonic wave velocities on different pre-incident rock samples, and converted those measurements to UPVC values using accepted equations and correlations. The ultrasonic wave velocity data yielded values that were somewhat lower than 6.35 microsips.

vi. Finally, I note that my estimated UPVC value of 6.35 microsips is roughly consistent with reported values of UPVC from other consolidated sandstones having similar ranges of porosity. My value of 6.35 microsips is, therefore, not unexpected, nor does it require any special explanation.

vii. A value of UPVC on the order of 12 microsips, which is used by some United States experts, is ***not*** consistent with, and is ***not*** supported by, any available data, including data from Weatherford's tests.

In the following section (Section 4), I provide an overview of the work that I performed to arrive at my opinions regarding UPVC, including a discussion of the work I did related to the three Weatherford tests, my literature review, and a comparison of my work to the United States experts. Section 5 then provides some detailed background and the mathematical and physical principals I used in my analysis. Sections 6 through 8 provide the detailed calculations I performed relating to each of the three Weatherford tests. Section 9 describes in further detail my findings in industry literature related to UPVC. Section 10 provides a brief conclusion to my report.

4. Overview of Work Performed

Based on my analysis of the relevant data, I conclude that the best estimate of the reservoir's average UPVC is 6.35 microsips over a range of pore pressures from 11,800 psi to 10,400 psi. 11,800 psi represents the highest pore pressure used by Weatherford in its uniaxial compression experiments, and it is close to the initial pore pressure of the Macondo reservoir. I have been advised that other experts for BP have estimated that the Macondo reservoir's final pore pressure was close to 10,400 psi. My estimate of the average UPVC remains relatively constant over a wide range of final pore pressures. For instance, I obtain nearly the same average UPVC over pore pressure ranging from 11,800 psi to 10,500 psi. Some United States experts use a final pressure less than 10,400 psi. Because UPVC is lower at lower pore pressures, my estimate of average UPVC would be lower than 6.35 microsips if I had used the final pressures used by some United States experts.

The following subsections describe the work I performed. Section 4.1 describes the data available for my evaluation. Section 4.2 provides general background on rock compressibility (more detailed background is provided in Section 5). Section 4.3 describes my analysis of uniaxial compression test data and the results I obtained (also described in more detail in Section 6). Section 4.4 describes my work involving "stairstep" porosity data (also described in more detail in Section 7). Section 4.5 covers the work I did related to ultrasonic velocity test data (also described in more detail in Section 8). Section 4.6 describes my findings in relevant literature regarding UPVC (with more description provided in Section 9). And finally, Section 4.7 compares my analysis to that of the experts for the United States.

By the end of Section 4, I hope to have provided a complete overview of the work I performed related to the Macondo reservoir. The subsequent sections provide additional detail about that work.

4.1 Samples, Testing, and Analysis Methods

While the Macondo well was being drilled, several dozen samples of reservoir rock were collected.[1] All of the rock samples from the Macondo reservoir were rotary sidewall core samples.[2] Rotary sidewall core samples are extracted using a tool that bores sideways into

---

[1] Jaime Loos Deposition, p. 49.

[2] Jaime Loos Deposition, p. 220.

the reservoir wall in order to extract cylindrical rock samples. The rock samples were sent to Weatherford Laboratories for evaluation and testing.

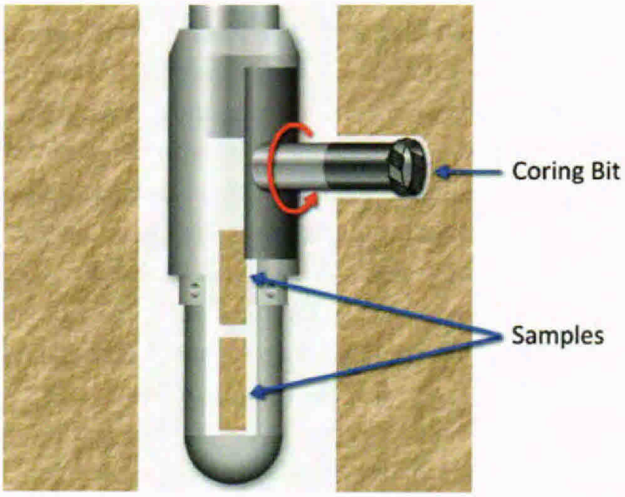


Fig. 4.1. Representative diagram of a rotary sidewall coring tool. The tool cuts cylindrical rock samples out of the wall of the wellbore. All of the rock samples from Macondo are rotary sidewall cores.

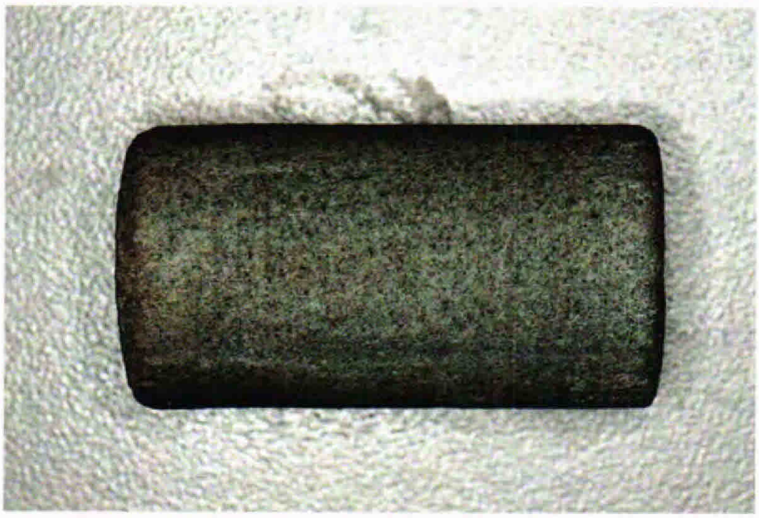


Fig. 4.2. Photograph of the Macondo reservoir rotary sidewall core rock sample number 3-16R.[3] Weatherford used sample number 3-16R and other cores for its uniaxial compression tests.

---

[3] Weatherford Photograph 18129.10_3-16R_wl_l.jpg, WFT-MDL-00039304.

It is common to perform compressibility testing on rotary sidewall core rock samples,[4] as well as to analyze and rely on the results of those tests.

For some reservoirs, laboratory data is available from "whole" or "conventional" core samples. Such samples are not available from the Macondo reservoir, but that does not affect my analysis. Although whole core rock samples are typically larger than rotary sidewall cores, I am not aware of any literature or information showing that the size of a sample is relevant to compressibility measurements. Indeed, rock mechanics theory suggests that the size of the sample should have no effect on the measurements of compressibility. Sometimes a rotary sidewall core sample can be so small that the strain gauges are affected by frictional end effects, but the effect of those frictional effects does not usually exceed a few percentage points.[5] Size — and specifically, length-to-diameter ratio — can be relevant to other types of rock tests, such as strength measurements, but it is not relevant to the tests that I analyze in this paper. The eight cores I evaluate look sufficiently intact to be suitable for the types of testing I analyze.

To reach my conclusions, I use data from the tests conducted by Weatherford Laboratories on the Macondo rock samples. Estimates of UPVC can be derived from several different types of laboratory measurements conducted on rock samples taken from wells. Here, Weatherford conducted three experiments on the Macondo samples that can be used to calculate UPVC: uniaxial compressibility tests; hydrostatic "stairstep" porosity tests, and ultrasonic velocity tests. Each type of test was conducted on a separate set of rock samples, and all three tests yield values that are in reasonable agreement with my opinion that the average UPVC of the Macondo reservoir is 6.35 microsips. None are at all consistent with a value as high as 12 microsips, a value used by some of the United States experts.

4.2 General Background on Rock Compressibility

The property of "compressibility" is, in general, a measure of the ability of a material to deform under pressure. Compressibility describes how much the rock volume changes for a given change in pressure, but there are various ways to look at the volume of a porous rock and the pressures acting on a porous rock.

---

[4] Weatherford also notes that it is common. Jaime Loos Deposition, p. 143.

[5] Jaeger *et al.*, 2007, section 6.3.

Rock volume: A porous rock is made up of solid mineral grains and empty pore space. Thus, an investigator can look at (1) the *mineral volume* of a porous rock — the volume of the solid mineral grains by themselves; (2) the *pore volume* of the porous rock — the volume of the empty pore space alone; or (3) the total *bulk volume* of the porous rock — the total volume of both the grains and the pores.

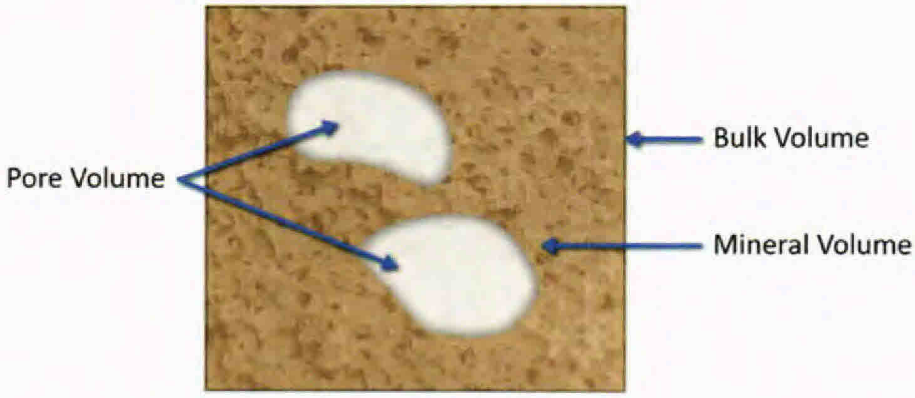


Fig. 4.3: Illustration of the "pore volume", "mineral volume", and "bulk volume" of a porous rock.

Rock pressure: A porous rock has internal *pore pressure* pushing outward due to the fluids inside the pore space, and the external *confining pressure* pushing inward due to gravity and other external forces. The confining pressure can be *lateral*, *i.e.*, from the sides, or *overburden*, *i.e.*, from the top and bottom. Moreover, the external confining pressures on a rock can be *hydrostatic* — the same in all directions; or *triaxial*— where the vertical stress is not the same as the horizontal stresses.

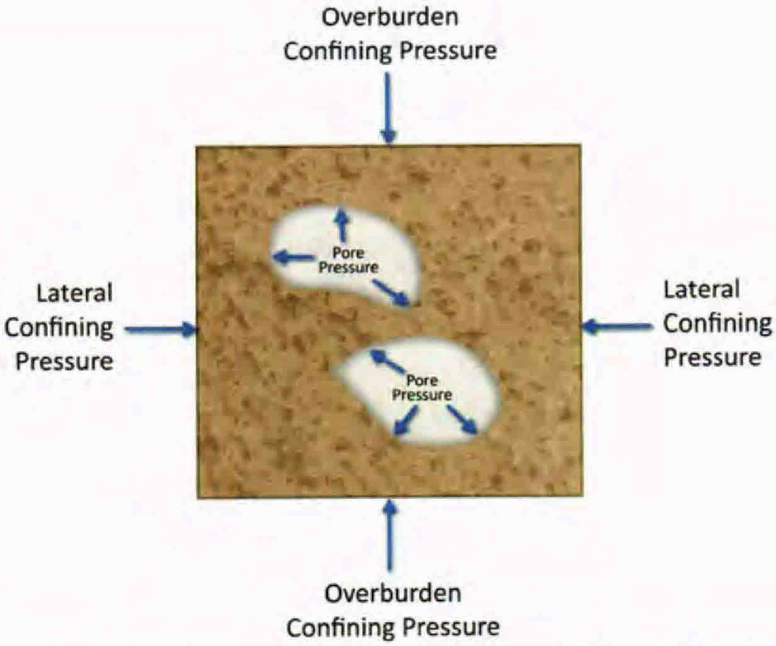


Figure 4.4: Illustration of the "pore pressure" and "confining pressure" that act on a porous rock.

Thus, one can evaluate, taking two examples, how the bulk volume changes with a change in the confining pressure under hydrostatic conditions, or how the pore volume changes with a change in the pore pressure under uniaxial strain conditions.

It is this latter type of compressibility — uniaxial pore volume compressibility, or UPVC — that is most relevant to reservoir modeling and material balance analysis. First, the primary interest with respect to volume is in the changing *pore* volume, not the changing mineral volume, because the pore volume defines how much fluid the porous rock can contain. Second, the primary interest on pressure is in the changing *pore* pressure, not the changing confining pressure, because removal of hydrocarbons from a reservoir will lower the pore pressure. Finally, the external pressures on a reservoir are different in the vertical and horizontal directions; when a reservoir is depleted and the pore pressure declines, the state of stress that acts on the rock is not hydrostatic. The vertical stress remains constant, since this stress is essentially due to the weight of the rock that lies above the reservoir. As the pore pressure declines, a rock would ordinarily contract laterally, but the reservoir rock is prevented from doing so by the vast expanse of rock to its sides. This type of deformation is known as uniaxial deformation, because the deformation occurs only in the vertical direction. Figure 4.5 illustrates uniaxial deformation.

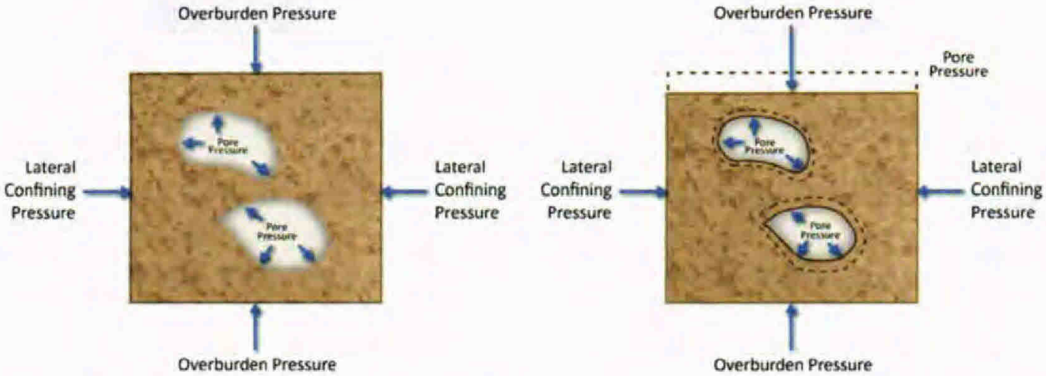


Fig. 4.5. Illustration of uniaxial deformation. The rock changes size (is deformed) along the vertical axis, but it does not change size along the horizontal axes. Dashed lines represent the original shape.

The different types of compressibility can be measured through different tests in laboratories. Then, using mathematical relations that can be derived between these compressibilities using the theory of elasticity, we can, for example, estimate the value of the pore volume compressibility from measurements of the bulk volume compressibility. The uniaxial pore compressibility is also related mathematically to the hydrostatic pore volume compressibility.

Section 5 of this report gives a more thorough overview of the theory of porous rock compressibility, as well as a description of the mathematical and physical relationships used in my work.

In this report, I have evaluated three different laboratory tests by Weatherford Laboratories, each measuring a different type of compressibility, and generated UPVC estimates based on each. The fundamental result was that each set of samples, and the measurements of each of the three rock properties, confirmed my estimate of the average Macondo UPVC as being 6.35 microsips or lower.

The following sections 4.2 through 4.4 provide a summary of the measurements and my analysis of those three tests.

4.3 First Calculation: Based on Uniaxial Compression Test Data — 6.35 microsips

The main set of data that I have used to estimate the average UPVC of the Macondo reservoir was collected by Weatherford during a set of uniaxial compression tests conducted on three samples from the Macondo well.[6] These data are the most representative measurements of UPVC available from the laboratory data, because they reproduce the sort of uniaxial deformation that occurs in the reservoir. It is these data that I use to develop my best estimate of the Macondo UPVC — 6.35 microsips.

In the uniaxial compressibility test, each rock sample was inserted into a testing apparatus, the pores were filled with pressurized fluid, and external pressure was applied to the top, bottom, and sides of the cylinder. Pore pressure was then reduced while the pressure on the sides of the sample was adjusted so that there was no change to the diameter of the sample. This mimics the uniaxial deformation the rock would experience in the reservoir, where the rock could compress and expand in the vertical direction, but the vast expanses of rock surrounding the sides of the reservoir prevent the reservoir rock from expanding or contracting laterally. For these tests, Weatherford used standard methods that are well accepted in the industry.

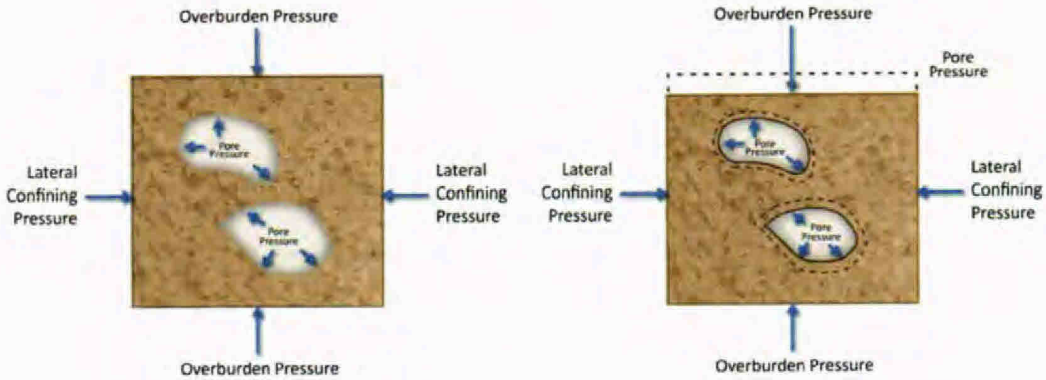


Fig. 4.6. Illustration of Weatherford's uniaxial compression test. The core sample is "loaded" with internal pore pressure, lateral confining pressure, and vertical overburden pressure (left diagram). Pore pressure is reduced while overburden pressure remains constant. The lateral confining pressure is adjusted so that the sample diameter remains constant (right diagram). The shrinkage of the core in the vertical direction is measured.

Although Weatherford calls these tests "uniaxial pore volume compressibility tests", the tests do not directly measure pore volume compressibility. Instead, the tests measure the change in *bulk* volume and yield values for the rock's uniaxial *bulk* volume compressibility. While the bulk volume compressibility measured in the test is closely related to the pore volume compressibility, it is slightly different. I converted the uniaxial

[6] Weatherford Pore Volume Compressibility Test, WFT-MDL-00082904.

bulk volume compressibility data to uniaxial *pore* volume compressibilities using accepted conversion equations (see Section 5 for those equations). Based on the Weatherford data, I determined that the average value of the UPVC, over the range of pressures from 11,800 psi down to 10,400 psi, was 8.57 microsips for sample 3-6R, 4.34 microsips for sample 3-16R, and 6.14 microsips for sample 3-22R. I then took the arithmetic average of the values from these three samples, which yielded 6.35 microsips. I consider this value to be the most accurate estimate of the Macondo UPVC, since it is based on data collected under conditions of uniaxial deformation.

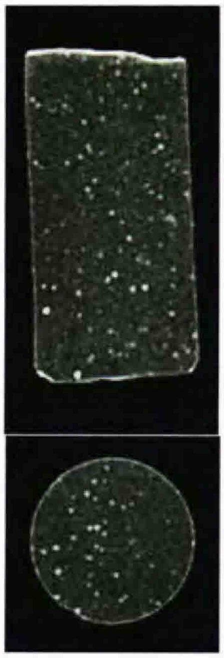
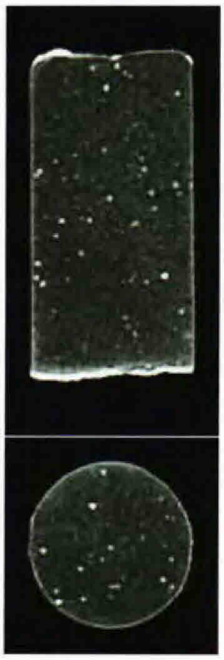
Sample Number:	3-6R	3-16R	3-22R
CT Scan Images:[7]			
Calculated UPVC:	8.57 microsips	4.34 microsips	6.14 microsips
Average of all samples:	**6.35 microsips**		

Table 4.1. Rock samples tested by Weatherford Laboratories for UPVC.

[7] Weatherford Photograph plug-18074_90-0.jpg, WFT-MDL-00039346; Weatherford Photograph plug-18129_10-0.jpg, WFT-MDL-00039352; Weatherford Photograph plug-18150_00-0.jpg, WFT-MDL-00039354.

A detailed description of the uniaxial compression test data, and my analysis of that data, can be found in Section 6.

4.4 Second Calculation: Based on Hydrostatic "Stairstep" Porosity Test Data — 5.47 microsips

The second set of data that I use to calculate UPVC comes from Weatherford's hydrostatic "stairstep" porosity test.[8] Weatherford conducted this test on three additional rock samples; the test provides confirmation of, and adds robustness to, the 6.35-microsip UPVC estimate that I determined from the uniaxial compression test data.

UPVC can be inferred from hydrostatic compression measurements. In the "stairstep" test, porosities were measured by Weatherford on three core samples: sample 3-8R, sample 3-21R, and sample 3-25R.[9] The porosity measurements were made by increasing the external confining pressure, while holding the pore pressure constant. They therefore correspond to a stress path in which the difference in pressure between the external confining pressure and the internal pore pressure (known as the "differential pressure" or "net confining stress") *increases* — which mimics the stress path that occurs in the reservoir during depletion. Converting the compressibility coefficient measured in these hydrostatic compression tests into the UPVC requires knowledge of a property known as the Poisson ratio. The Poisson ratio of a consolidated sandstone usually lies between 0.1 and 0.2 (see Table 5.1), which I have used in my calculations. (And, in fact, the two values measured during ultrasonic tests on two other Macondo cores were 0.13 and 0.18, as will be discussed further in Section 8.)

Using the hydrostatic porosity measurements, I calculate an average UPVC of between 4.56 and 5.47 microsips at a differential pressure of 2372 psi, which is the mean value of the differential pressure experienced by the reservoir over the relevant range of pore pressure. The UPVC values derived from the hydrostatic compression tests is therefore roughly consistent with the value of 6.35 microsips that was calculated from the uniaxial compression tests. It is, once again, not consistent with a UPVC value of 12 microsips.

Since the stresses in the hydrostatic "stairstep" porosity test were applied in all directions, this test essentially provides a measure of the average compressibility in all three

---

[8] Weatherford Laboratories Report WFT Labs HH-46949, WFT-MDL-00129171.

[9] Weatherford Laboratories Report WFT Labs HH-46949, WFT-MDL-00129171.

directions. The fact that the UPVC values computed from these tests were close to the value estimated from the uniaxial tests implies that the samples were not highly anisotropic — they had the same compressibility values in all directions. This provides further justification of the use of rotary sidewall core rock samples.

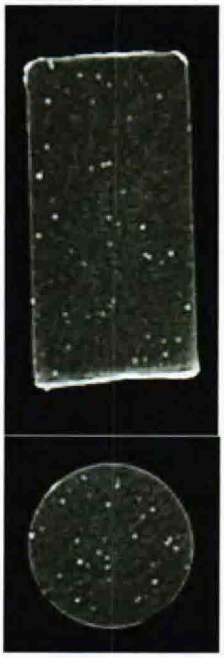
Sample Number:	3-8R	3-21R	3-25R
CT Scan Images:[10]			
Calculated UPVC (average of all samples):	**4.56 to 5.47 microsips**		

Table 4.2. Rock samples tested by Weatherford Laboratories for hydrostatic "stairstep" porosity.

I discuss my analysis of the hydrostatic "stairstep" porosity measurements further in Section 7.

---

[10] Weatherford Photograph plug-18081_80-0.jpg, WFT-MDL-00039236; Weatherford Photograph plug-18147_90-0.jpg, WFT-MDL-00039245; Weatherford Photograph plug-18161_00-0.jpg, WFT-MDL-00039356.

4.5 Third Calculation: Based on Ultrasonic Velocity Test Data — 4 microsips

The third and final set of Weatherford data I use is from ultrasonic velocity measurements.[11] The result of this analysis is an estimated UPVC of around 4 microsips. This once again confirms that the best estimate of Macondo's average UPVC is no more than 6.35 microsips and is not anywhere near 12 microsips.

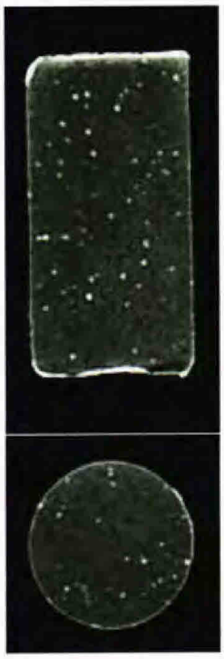
Sample Number:	3-17R	3-19R
CT Scan Images:[12]		
Calculated UPVC:	4 microsips	4 microsips
Average of all samples:	**4 microsips**	

Table 4.3. Rock samples tested by Weatherford Laboratories for ultrasonic velocities.

Weatherford measured ultrasonic wave velocities on two dry cores from the Macondo reservoir — cores that were not previously tested in either the uniaxial compressibility tests, or in the hydrostatic porosity tests. The bulk volume compressibility influences the

[11] Weatherford Laboratories Rock Mechanics Final Report (Acoustic Velocities / Mohr-Coulomb Failure Analysis), Weatherford Laboratories Report WFT Labs HH-46949, WFT-MDL-00082902.

[12] Weatherford Photograph plug-18131_90-0.jpg, WFT-MDL-00039242; Weatherford Photograph plug-18141_90-0.jpg, WFT-MDL-00039353.

speed at which acoustic waves travel through a rock. If the speeds of the waves are measured on a dry rock, the hydrostatic bulk volume compressibility can be calculated. Since the bulk and pore compressibilities are related to each other through mathematical relations, ultrasonic measurements can also be used to provide an indirect way to estimate UPVC.

Weatherford directly converted the measured wave velocities to a *dynamic bulk modulus*. From that information, I was able to use standard rock physics equations to calculate a *dynamic* UPVC. The resulting value, for both of the samples, was 1.15 microsips. While this value is much lower than the values estimated from the other two measurements, it is known that *dynamic* compressibilities (like the ones here, based on the dynamic bulk modulus) are typically lower than *static* compressibilities (like the one needed for material balance and reservoir simulation). Known handbook correlations reveal that the conversion of the dynamic bulk modulus to the static bulk modulus requires an increase by about a factor of three — yielding a *static* UPVC of about 4 microsips. Although this calculation is not as precise as the other methods, it can nevertheless be concluded that the UPVCs that can be derived from the ultrasonic measurements are roughly consistent with the value of 6.35 microsips that was calculated from the uniaxial compression tests, but are *not* consistent with values of UPVC as large as 12 microsips.

Further exposition of my ultrasonic wave analysis can be found below in Section 8.

4.6 Literature Review Supports Measured Data

The Macondo average UPVC estimate of 6.35 microsips is roughly consistent with other sets of measured values of the UPVC of consolidated sandstones having porosities of about 20% — like the Macondo reservoir. For example, the classic 1953 correlation developed by Hall indicates that consolidated sandstones of 20% to 23% porosity would have pore compressibilities of about 3 to 4 microsips;[13] the porosity of the Macondo reservoir is in this porosity range. Another well-known data set collected by Newman in 1973, which consisted of almost 100 consolidated sandstones, showed that all samples with porosities greater than 20% had pore compressibilities less than 6 microsips.[14] The estimated UPVC of 6.35 microsips, which is based on laboratory measurements on the Macondo rock samples, is therefore neither unexpected, nor does it require any special

[13] Hall, 1953.

[14] Newman, 1973.

explanation. On the other hand, a UPVC of 12 microsips for a consolidated sandstone of about 20% porosity would be anomalous, and inconsistent with known values from the petroleum engineering literature.

The literature, which supports my numerical conclusions, is discussed further in Section 9.

4.7 Comparison to the Work of the United States Experts

I have reviewed the laboratory measurements by Weatherford Laboratories, and applied my expertise in rock mechanics to conclude that the best estimate of Macondo reservoir UPVC is 6.35 microsips.

While none of the United States experts has presented a detailed analysis of the compressibility data, three of the experts do employ reservoir analysis techniques that use values for the Macondo reservoir's UPVC as an input:

- Dr. Mehran Pooladi-Darvish: In his reservoir analysis, Dr. Pooladi-Darvish uses a "base estimate" of 6 microsips for formation compressibility (*i.e.*, UPVC), which is consistent with my opinion regarding the Macondo reservoir's UPVC. According to his report, this "base estimate" was "taken from [the] uniaxial strain pore volume compressibility test conducted by Weatherford" — the same tests that I reviewed for my best estimate of Macondo UPVC. Dr. Pooladi-Darvish is the only United States expert who references the Weatherford Laboratories data in his expert report.[15]
- Dr. Mohan Kelkar & Dr. Rajagopal Raghavan: The report of Drs. Kelkar and Raghavan includes a material balance analysis that uses a base UPVC value of 12 microsips as one of its inputs. Citing a July 8, 2010 PowerPoint presentation by Robert Merrill, Drs. Kelkar and Raghavan claim "this is the most likely value of formation compressibility...."[16] However, they have not cited or referenced any laboratory data to support this conclusion. Nor does their report include any analysis of available compressibility data. Their conclusion that 12 microsips is "the most likely value of formation compressibility" is not supported by the laboratory and other data that I have reviewed.

---

[15] Pooladi-Darvish Report Appendix III, page 8. Dr. Pooladi-Darvish also asserts that 12 microsips is "taken from [the] uniaxial strain pore volume compressibility test conducted by Weatherford", and cites the Weatherford uniaxial compression test summary (BP-HZN-2179MDL02394184). I have reviewed this data, and it does not support a UPVC of 12 microsips under any pressure conditions (see Section 6).

[16] Kelkar/Raghavan Report, page 28.

In 2010, Dr. Kelkar issued a report commissioned by the Flow Rate Technical Group (FRTG) wherein he stated that, "from the available data", the base case for average rock compressibility at a reference pressure of 11,000 psi was 5.61 microsips.[17] This value for UPVC is very similar to the value I have calculated based on the available data.

- Dr. Paul Hsieh: Dr. Hsieh uses an assumed "effective formation (or pore) compressibility, c_f" (*i.e.*, UPVC) of 12 microsips.[18] Dr. Hsieh states that he "did not look at the data" from Weatherford Laboratories, and did not do his own analysis of UPVC,[19] but rather used a value of UPVC that he says was provided to him by Kelly McAughan and Bob Merrill. However, as mentioned above, this value of twelve microsips is not consistent with any of the data I have reviewed.

Therefore, no United States expert has presented data or analysis that contradicts my analysis of the Weatherford Laboratory measurements. Furthermore, the one United States expert who has analyzed the data agrees with my results.

---

[17] Kelkar Modeling Report (2010), Deposition Exhibit 9859, page 17.

[18] Hsieh Pre-Decisional Draft Report, Deposition Exhibit 8615, page 12; Paul Hsieh Deposition, p. 264.

[19] Paul Hsieh Deposition, p. 354.

5. Basic Theory of Porous Rock Compressibility

There are many ways to describe the compressibility of a porous rock, all of which are related by known mathematical formulae. Ultimately, the value needed for reservoir engineering is what is called the uniaxial pore volume compressibility (UPVC). I begin this section with a discussion of the units commonly used in the field of rock mechanics (Section 5.1). Next I describe the hydrostatic porous rock compressibilities (Section 5.2) because they form the foundation for understanding porous rock compressibilities. I then describe uniaxial compressibilities and how they relate to hydrostatic compressibilities (Section 5.3). And finally in this section, because my analysis partly relies on ultrasonic velocity measurements, I describe the relationship between those velocities and rock compressibility (Section 5.4).

5.1 Units of Measurement of the Compressibilities

The oil and gas industry in the United States generally measures distances in units of feet and inches and measures pressure and stresses in units of psi, or "pounds per square inch". Since every compressibility parameter represents the fractional change in volume due to an incremental change in pressure, the compressibilities have units of 1/psi. But reservoir sandstones are relatively stiff, in the sense that pressure increments on the order of thousands of psi would be needed to cause a 1% change in pore volume, and so hydrostatic pore volume compressibilities of sandstones usually have magnitudes in the range of 1×10^{-6}/psi to 30×10^{-6}/psi.[20] (Note that this upper range is observed in unconsolidated sandstones having porosities much greater than 20%, and which are not analogous to the Macondo samples.) To avoid frequent mention of "ten to the minus six power", the unit of 1×10^{-6}/psi is often referred to as a "microsip", where "micro" is the standard terminology for 10^{-6} (one part in one million), and "sip" indicates the inverse of a psi. Therefore, for example, a pore compressibility value of 6×10^{-6}/psi would be referred to as "six microsips".

5.2 Hydrostatic Porous Rock Compressibilities

"Compressibility" is the material property that quantifies the relationship between the stress (or pressure) that acts on a body, and the resulting fractional change in the volume of that body. For a solid, non-porous body, the compressibility C is defined as

[20] Newman, 1973; Crawford *et al.*, 2011.

$$C = \frac{-1}{V}\left(\frac{dV}{dP}\right), \qquad \text{(eq. 5.1)}$$

where V is the volume, P is the pressure to which the body is subjected, and the term in parentheses is the derivative of the volume with respect to pressure, *i.e.*, the incremental change in volume divided by the incremental change in pressure. Since an increase in the external pressure will cause the volume to decrease, the minus sign in equation 5.1 causes the defined compressibility to be a positive number.

The rocks in petroleum reservoirs are, however, composed of various mineral grains — they are not completely solid. These rocks are porous; they contain cracks and pores that are filled with oil, gas, and water, to varying degrees. Three different types of volumes can be defined for a porous rock (Fig. 5.1).

- *Bulk volume* (V_b): This is the total volume, counting both the volume of the grains and the volume of the empty pore space. The bulk volume is the volume enclosed by the square in Figure 5.1.
- *Mineral (or Grain) Volume* (V_m): This is the volume occupied by the mineral grains. The mineral volume is indicated by the brown region in Figure 5.1.
- *Pore Volume* (V_p): This is the portion of the bulk volume that is not occupied by mineral grains. The pore volume is represented by the white regions in Figure 5.1.

It is, by definition, always the case that $V_b = V_m + V_p$. The fraction of the total bulk volume that consists of pore space is known as the porosity, ϕ, and is defined by $\phi = V_p / V_b$.

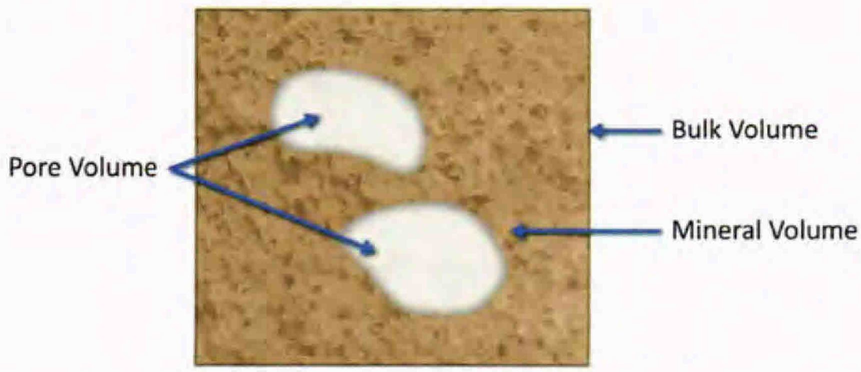


Fig. 5.1. A representative porous rock diagram, showing the bulk volume (region bounded by the closed outer solid square), the pore volume (the white regions), and the mineral volume (brown region).

A reservoir is subjected to external vertical stresses that are due to gravity (*i.e.*, the weight of the rock that lies above the reservoir, the so-called "overburden"), and external lateral stresses. Being porous, reservoir rock is also subjected to internal pressures that act on the walls of the pores, due to the pressure caused by fluid in the pores. The external stresses tend to compress both the bulk volume and the pore space, whereas the pore pressure tends to cause both the bulk volume and the pore space to expand. During depletion of a reservoir, however, as the pore pressure decreases, the pore volume will contract (like a car tire that has a leak, for example). The total bulk volume change undergone by a rock is equal to the volume change of the pore space, plus the volume change of the mineral grains.

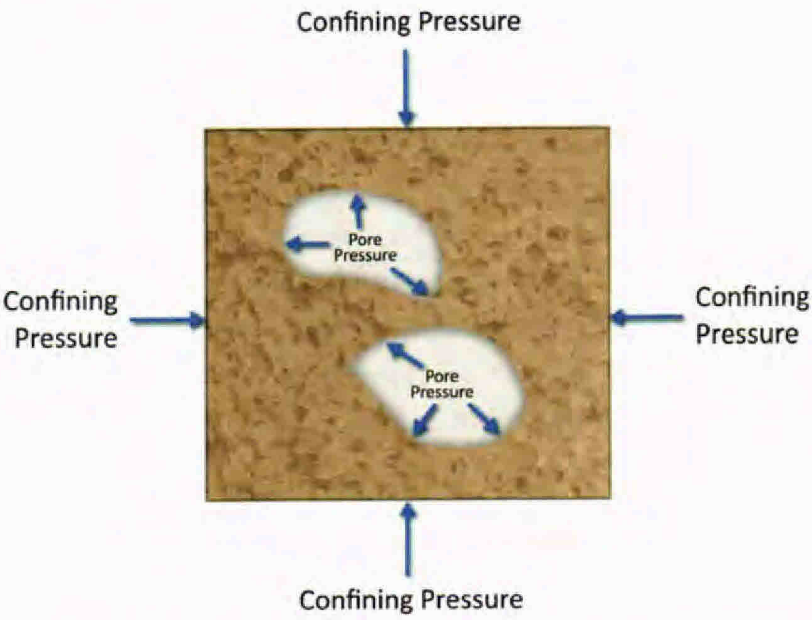


Figure 5.2. Porous rock subjected to an external "confining" pressure and an internal "pore" pressure that acts over the surfaces of its internal pore walls.

Figure 5.2 shows a generic piece of a porous rock, acted upon by a pore pressure, P_p, and an external confining pressure, P_c. In this particular type of stress state, known as "hydrostatic compression", the vertical and lateral pressures are equal. Although this simplified state of stress is not generally the one that exists in the reservoir, it is the starting point for the theory of porous rock behavior, and it is often used in laboratory measurements, as described in Section 7.

Because $V_b = V_m + V_p$, there are two independent mathematical volume variables during hydrostatic compression — the bulk volume (V_b), and the pore volume (V_p) — and two independent mathematical pressure variables — the confining pressure (P_c), and the pore pressure (P_p). Consequently, four different hydrostatic porous rock compressibilities can be defined.[21] Each of these compressibilities is defined as the fractional change in a volume, divided by the change in a pressure. Mathematically, these definitions are expressed in terms of partial derivatives, as follows:

$$C_{bc} = \frac{-1}{V_b}\left(\frac{\partial V_b}{\partial P_c}\right)_{P_p}, \qquad \text{(eq. 5.2)}$$

$$C_{bp} = \frac{1}{V_p}\left(\frac{\partial V_b}{\partial P_p}\right)_{P_c}, \qquad \text{(eq. 5.3)}$$

$$C_{pc} = \frac{-1}{V_p}\left(\frac{\partial V_p}{\partial P_c}\right)_{P_p}, \qquad \text{(eq. 5.4)}$$

$$C_{pp} = \frac{1}{V_b}\left(\frac{\partial V_p}{\partial P_p}\right)_{P_c}, \qquad \text{(eq. 5.5)}$$

where the pressure outside the parentheses indicates that that pressure must be held constant as the other pressure changes. Since an increase in the confining pressure would cause the bulk and pore volumes to decrease, whereas an increase in the pore pressure would cause these volumes to increase, minus signs are needed in two of these definitions in order to ensure that all four of these porous rock compressibilities are positive numbers.

The first "bulk compressibility", defined by equation 5.2, namely C_{bc}, is the fractional change in the bulk volume caused by an increase in confining pressure. This compressibility is directly analogous to the compressibility of a non-porous material, as defined by equation 5.1. This compressibility is relevant to seismic or acoustic wave propagation since, by definition $C_{bc} = 1/K$, where K is the bulk modulus, which is one of the main parameters that controls the speed of compressional waves (P-waves). (The compressibility is most often used when discussing compressibility measurements, whereas ultrasonic measurements are typically discussed in terms of the bulk modulus;

---

[21] Zimmerman, 1991.

these parameters are essentially equivalent, as both C_{bc} and $1/K$ are defined by eq. 5.2.) I use the relationship between C_{bc} and UPVC in Section 8.

The other bulk compressibility, C_{bp}, defined by equation 5.3, represents the fractional change in bulk volume that would be caused by a change in the pore pressure. This compressibility is useful in surface subsidence calculations, as explained by Geertsma.[22] A closely related parameter, namely the uniaxial version of C_{bp}, is used in my analysis of the Weatherford Laboratories uniaxial compression data in Section 6.

The two compressibilities defined in equations 5.4 and 5.5, C_{pc} and C_{pp}, are *pore compressibilities*. Hall referred to C_{pc}, which represents the fractional change in pore volume due to a change in confining pressure, as the "formation compaction" coefficient.[23] C_{pc} is not used in my analysis of the Weatherford data.

The pore compressibility defined in equation 5.5, which is the fractional change in pore volume due to a change in pore pressure, was denoted by Hall as "effective rock compressibility"[24] The pore compressibility C_{pp} is used extensively in reservoir engineering, since it is related to the volume of pore fluid that is released from a rock as the pore pressure declines. It appears directly in the material balance equations that are used to calculate oil and gas reserves.[25] This compressibility, or more accurately a close relative thereof (specifically, the *uniaxial* pore volume compressibility that is relevant to depletion that occurs under conditions in which the rock is not allowed to expand or contract laterally, as explained further below), is added to the compressibility of the reservoir fluid to give the "total compressibility" term that appears in the basic equation that governs reservoir pressure analysis and transient flow to a well.[26] The *uniaxial* version of C_{pp} is referred to throughout this report as *uniaxial pore volume compressibility*, or UPVC.

Under the standard assumption that the mineral grains behave as an elastic material, which means that the incremental volume change undergone by the grains is directly proportional to the incremental change in pressure, it can be shown mathematically that

---

[22] Geertsma, 1973.

[23] Hall, 1953.

[24] Ibid.

[25] Dake, 1978.

[26] Matthews and Russell, 1967.

the four hydrostatic porous rock compressibilities are related to one another by the following three equations:[27]

$$C_{pc} = C_{pp} + C_m, \quad \text{(eq. 5.6)}$$

$$C_{bp} = \phi(C_{pp} + C_m), \quad \text{(eq. 5.7)}$$

$$C_{bc} = C_m + \phi(C_{pp} + C_m), \quad \text{(eq. 5.8)}$$

where C_m is the compressibility of the mineral grains. These relations allow one to infer the numerical value of one of the porous rock compressibilities, based on measurements of one of the other compressibilities. In fact, although C_{pp} is the most important porous rock compressibility in petroleum reservoir engineering, it is the most difficult to directly measure. Consequently, the usual practice is to measure one of the other compressibilities and use equations such as 5.6 through 5.8 to infer the value of C_{pp}. This procedure will be applied in Sections 6 through 8 to the Weatherford data.

The mineral compressibility, C_m, that appears in equations 5.6 through 5.8, is essentially constant for a given rock, and does not vary with pressure. The porosity, ϕ, that appears in equations 5.7 and 5.8 represents the initial porosity that exists at the start of the depletion process, and so will be constant as depletion proceeds. But the four porous rock compressibility coefficients, as defined in equations 5.2 through 5.5, are generally pressure-dependent, particularly in sandstones. These coefficients vary as functions of the differential pressure, which is defined as the difference between the confining pressure and the pore pressure. These compressibilities are often quite high at low values of the differential pressure, but decrease as the differential pressure increases, leveling off to some nearly constant value as the differential pressure reaches a few thousand psi.

In particular, during depletion of the Macondo reservoir, the pore pressure decreased. The differential pressure — the difference between the confining pressure and the pore pressure — will therefore increase. Because the pore compressibility *decreases* as the differential pressure *increases*, it is to be expected that the pore compressibility will decrease during depletion. However, the extent to which it decreases will depend on the pore geometry of the reservoir rock[28] and also on the range of the change in the differential pressure.

---

[27] Geertsma, 1957; Zimmerman, 1991.

[28] Zimmerman, 1991.

One of the earliest measurements of the hydrostatic pore compressibility of a reservoir rock was made by Carpenter and Spencer[29] on a Frio sandstone from East Texas, which had an initial porosity of 30%. Figure 5.3 shows the fractional change in pore volume as a function of differential pressure, and Figure 5.4 shows the calculated pore compressibility, C_{pc}, as plotted by Zimmerman,[30] from the data shown in Figure 5.3.

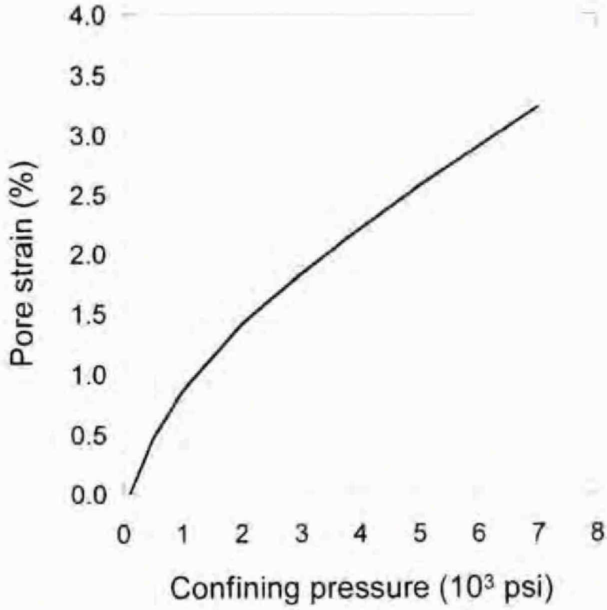


Fig. 5.3. Pore strain of a Frio sandstone from East Texas, as a function of confining pressure, measured at zero pore pressure.[31]

---

[29] Carpenter and Spencer, 1940.

[30] Zimmerman, 1991.

[31] Carpenter and Spencer, 1940.

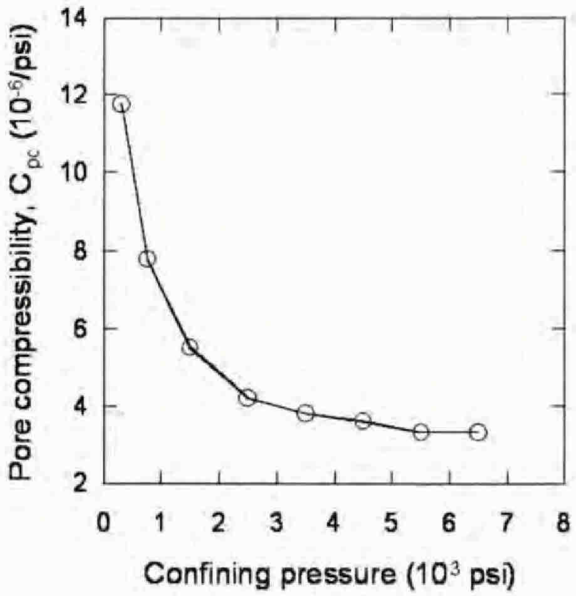


Fig. 5.4. Pore compressibility of a Frio sandstone from East Texas, as calculated by Zimmerman,[32] based on the data shown in Fig. 5.3.

The curve in Figure 5.4 is typical, in both its general trend and its numerical values, of most consolidated sandstones. Note that the differential pressure in the Macondo reservoir during its period of depletion was in the range of about 2000-3000 psi.

5.3 Uniaxial Pore Volume Compressibility

The hydrostatic porous rock compressibilities defined in Section 5.2, above, are relevant to situations in which the entire outer boundary of the rock is subjected to a confining pressure of the same magnitude. In particular, the pore compressibility C_{pp} is directly applicable to the situation in which the external confining pressure remains constant as the pore pressure changes. But when fluid is withdrawn from a reservoir, it is not realistic to assume that, as the pore pressure declines, the external confining pressure remains constant. Although the pore pressure initially declines only within a region of the reservoir around the well, the large expanse of reservoir rock farther away from the well experiences no change in pore pressure, and this rock would prevent the rock within the depleted region from expanding or contracting laterally. Later in time, as the entire reservoir experiences a decrease in pore pressure, the rock outside the reservoir would

[32] Zimmerman, 1991.

provide the same lateral restraint on compression or expansion of the reservoir rock. Consequently, although the vertical stress, which is essentially due to the weight of the rock layers above, remains constant during reservoir depletion, the reservoir rock deforms under conditions in which no lateral deformation (or "lateral strain") is permitted.[33] This type of deformation is known as "uniaxial deformation", since the rock is free to contract or expand only along the vertical axis (Fig. 5.5).

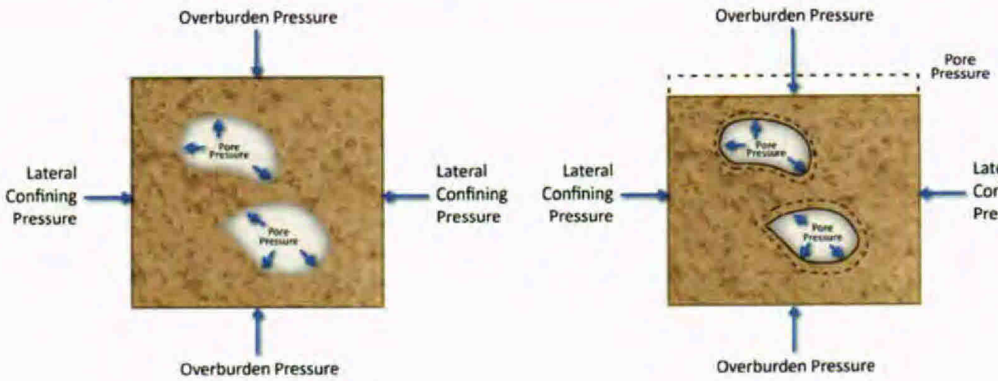


Fig. 5.5. Porous rock undergoing uniaxial deformation due to pore pressure depletion. Left: state of stress and deformation before the start of depletion. Right: As the pore pressure decreases, the vertical stress remains constant, which cause vertical strain. The lateral confining stresses, however, continually adjust themselves so that there is no lateral strain. Hence, the pore volume decreases, and the rock compresses vertically. But the rock does not compress or expand laterally. Dashed lines represent the *original* geometry.

The pore compressibility that is most relevant to reservoir depletion is therefore the *uniaxial pore volume compressibility*, which can be defined as

$$UPVC \equiv C_{pp}^{uni} = \frac{1}{V_p}\left(\frac{\partial V_p}{\partial P_p}\right)_{\varepsilon_{xx},\varepsilon_{yy},\tau_{zz}}, \qquad \text{(eq. 5.9)}$$

where the terms outside of the parenthesis indicate that the vertical stress, τ_{zz}, and the lateral strains, ε_{xx} and ε_{yy}, each remain constant as the pore pressure declines.

Since the rock is constrained against deforming laterally (*i.e.*, horizontally) during uniaxial compression or expansion, it is to be expected that the UPVC, C_{pp}^{uni}, will be less than the hydrostatic pore volume compressibility, C_{pp}.

[33] Fjaer *et al.*, 2008.

The precise relationship between these two compressibility coefficients can be derived using the full three-dimensional equations of poroelasticity.[34] The relationship is:[35]

$$C_{pp}^{uni} = C_{pp} - \frac{2(1-2\nu)\alpha}{3(1-\nu)}(C_{pp} + C_m) \qquad \text{(eq. 5.10)}$$

In this equation, the parameter ν is known as the Poisson ratio, and represents the ratio of lateral strain to axial (vertical) strain when the rock is acted upon by a uniaxial stress. The parameter α is the *Biot coefficient*,[36] which is defined as $\alpha = 1-(C_m / C_{bc})$. The Biot coefficient represents the fraction of the total bulk volume change that takes place in the pore space, as opposed to taking place in the mineral grains, during hydrostatic compression. The Macondo reservoir's burial depth, confining pressures, and lab-measured "unconfined compressive strength" all confirm that the Macondo reservoir consists of "consolidated" sandstone (see Section 9). The Poisson ratio of such consolidated sandstones will usually lie in the range of 0.1 to 0.2, and the Biot coefficient will usually lie in the range of 0.6 to 0.9.[37] The hydrostatic pore compressibility, C_{pp}, is usually an order of magnitude larger than the mineral compressibility, C_m.

Table 5.1 shows some typical data for six different consolidated sandstones, collated from various sources by me in a paper I authored in 2000.[38] The porosities (ϕ) of these six sandstones, shown in the second column, range from .02 to .26 (2% to 26%). The Poisson ratios (ν) of these sandstones, shown in column 3, lie in the range of 0.12 to 0.20. The Biot coefficients (α) of these sandstones, shown in column 4, lie in the range of 0.64 to 0.85. The ratios of uniaxial to hydrostatic pore volume compressibility, as computed from equation 5.10, are shown in the last column; these ratios range from 0.51 to 0.64. Thus, the numerical value of the UPVC is usually slightly more than one-half of the hydrostatic pore volume compressibility.[39]

---

[34] Zimmerman, 2000a.

[35] Zimmerman, 2000b, 2000c; Fjaer *et al.*, 2008.

[36] Biot, 1941.

[37] Detournay and Cheng, 1993; Jaeger *et al.*, 2007.

[38] Zimmerman, 2000c.

[39] Ibid.

Sandstone	ϕ	ν	α	$C_{pp}^{uni} / C_{pp}^{hydro}$
Ruhr sandstone	.02	.12	.65	.62
Berea sandstone	.19	.20	.79	.59
Weber sandstone	.06	.15	.64	.64
Ohio sandstone	.19	.18	.74	.59
Boise sandstone	.26	.15	.85	.51
Pecos sandstone	.20	.16	.83	.53

Table 5.1. Ratio of uniaxial to hydrostatic pore volume compressibility in some consolidated sandstones.[40]

I am able to use the relationships between hydrostatic and uniaxial compressibilities in Section 7 to convert Weatherford's measurements of hydrostatic compressibilities to estimates of UPVC.

5.4 Relationship between Pore Compressibility and Ultrasonic Velocities

Measurements of ultrasonic velocities can also be used to assess pore volume compressibility. UPVC is essentially a measure of the stiffness of the rock. As such, it is related to other stiffness and compressibility parameters. For example, as discussed in Section 5.3, the UPVC is related to the hydrostatic pore volume compressibility through equation 5.10. The hydrostatic pore volume compressibility, C_{pp}, is in turn (and as shown in Section 5.2) related to the bulk compressibility, C_{bc}, through equation 5.8. The inverse of the bulk compressibility is, by definition, equal to the bulk modulus, K, *i.e.*, $K = 1/C_{bc}$.[41] Since the bulk modulus K can be measured via ultrasonic wave measurements, and the UPVC can be related to C_{bc} using equations 5.8 and 5.10, the UPVC can be estimated from ultrasonic measurements, as will be described in Section 8.

The bulk modulus, K, along with the shear modulus, G, are the two stiffness parameters that control the bulk deformation of a rock. The bulk modulus represents the stiffness of the rock when it is compressed hydrostatically from all sides (Fig. 5.6, left), in which case the volume of the rock changes, but its shape does not change. The shear modulus, on the other hand, represents the stiffness of the rock when it is compressed in one direction and extended in the other direction (Fig. 5.6, right), in which case the shape of

[40] Ibid.

[41] Zimmerman, 1991.

the rock changes, but its volume does not change. The bulk and shear moduli are related to each other through the equation $G = 3K(1-2\nu)/2(1+\nu)$, where ν is the Poisson ratio.[42]

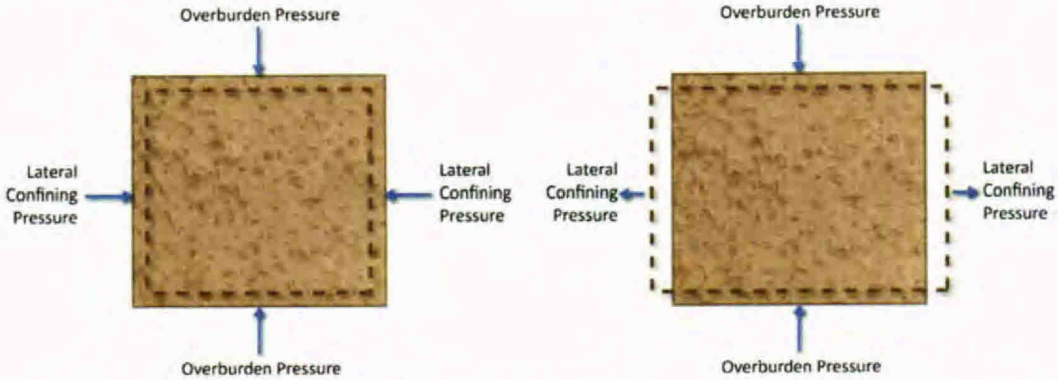


Fig. 5.6. Left: Representation of a rock subjected to compression in the vertical and horizontal directions. Right: Representation of a rock subjected to compression in one direction and extension in the other direction. In both figures, the solid line represents the shape of the rock *before* the pressures are applied, and the dashed line represents the shape of the rock *after* the pressures are applied.

These two stiffness parameters, which are also known as *elastic moduli*, control the speed of the two types of seismic/ultrasonic waves that can travel through a rock: the compressional wave, called the "P-wave", and the shear wave, called the "S-wave". The speeds of these two waves are given by[43]

$$V_P = \sqrt{\frac{K + (4/3)G}{\rho}}, \qquad \text{(eq. 5.11)}$$

$$V_S = \sqrt{\frac{G}{\rho}}, \qquad \text{(eq. 5.12)}$$

where ρ is the density of the rock. According to these equations, waves travel faster in a stiffer rock than in a softer rock, and travel more slowly in a denser rock than in a less dense rock.

In a reservoir, the pores of the rock will be filled with fluid. The pore fluid has the effect of stiffening the rock, *i.e.*, increasing the bulk modulus, *K*, and also increases the density

---

[42] Jaeger *et al.*, 2007; Mavko *et al.*, 2009.

[43] Mavko *et al.*, 2009, p. 81.

of the rock.[44] However, for measurements that are made in a laboratory on a dry rock, the various effects related to the pore fluid are not relevant. If the P-wave and S-wave velocities are measured on a dry rock, the bulk modulus can be found from equations 5.11 and 5.12:

$$K = \rho\left(V_P^2 - \frac{4}{3}V_S^2\right), \qquad \text{(eq. 5.13)}$$

after which C_{bc}, which is related by definition to K through $K = 1/C_{bc}$, can be related to the uniaxial pore compressibility through equations 5.8 and 5.10.

[44] Jaeger *et al.*, 2007; Mavko *et al.*, 2009.

6. Uniaxial Compression Measurements on Macondo Samples

Weatherford analyzed three pre-incident samples of Macondo reservoir rock using a set of uniaxial compression tests.[45] Because these measurements were made under conditions of uniaxial strain, these tests supply the data that is most directly relevant to the calculation of the UPVC.

The general procedures for the so-called "uniaxial strain pore volume compressibility test" are described by Weatherford,[46] and are summarized here briefly. Cylindrical specimens were saturated with kerosene, and then placed between two end-caps, and a heat-shrink jacket placed over the specimen to separate the pore fluid from the pressurized fluid that is used to apply the lateral confining pressure. Axial strain and radial strain devices were mounted in the end-caps, and on the lateral surface of the specimen, respectively. The lateral confining pressure was first brought to the initial reservoir value of 13,300 psi.[47] The pore fluid pressure was brought to the initial reservoir value of 11,800 psi. The axial stress was then increased to the initial reservoir overburden stress of 14,800 psi.[48] After establishing the initial reservoir stress state, all pressures were maintained constant for a sufficient time to allow the stresses and strains to stabilize.

The pore pressure was then reduced at a rate of 0.5 psi per second, and pressure and displacement data were recorded at fixed time intervals. During the pore pressure depletion, the lateral pressure was continually adjusted to maintain zero radial strain, *i.e.*, making sure there was only uniaxial deformation. The overburden stress was maintained constant throughout the test, at 14,800 psi. The test ended when the pore pressure had been reduced to 3800 psi. The method used by Weatherford was a standard and accepted approach in the oil and gas industry. Because the gauges used to take measurements of pressure and strain are generally accurate to within a few percent, the accuracy of the raw data collected in these tests is very high. Precise quantification of the uncertainty in the

---

[45] Weatherford Laboratories Rock Mechanics Final Report (Uniaxial Pore Volume Compressibility Tests), Weatherford Laboratories Report WFT Labs HH-46949, WFT-MDL-00130933.

[46] Ibid.

[47] Jaime Loos Deposition, pages 138-9.

[48] Ibid.

computed UPVC is difficult, but this uncertainty is much less than the factor of two that would be needed in order to reconcile the data with a claimed UPVC of 12 microsips.

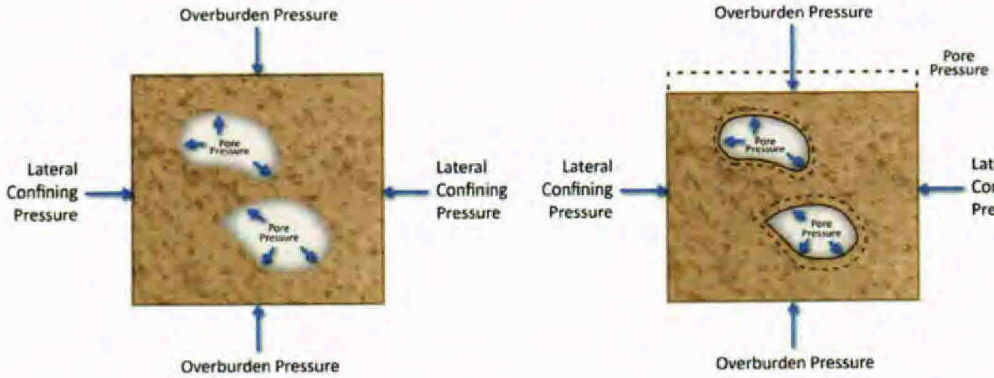


Figure 6.1. Illustration of Weatherford's uniaxial compression test. The core sample is "loaded" with internal pore pressure, lateral confining pressure, and vertical overburden pressure (left diagram). Pore pressure is reduced while overburden pressure remains constant. The lateral confining pressure is adjusted so that the sample diameter remains constant (right diagram). The shrinkage of the core in the vertical direction is measured.

It is important to note that, in these measurements, the *pore volume* was not actually monitored. Rather, changes in the *bulk volume* were measured, and in the original data analysis, "it [was] assumed that the grain compressibility is negligible and hence the change in the pore volume (ΔV_p) is equal to the change of bulk volume (ΔV_b)".[49] In other words, the change in volume that occurred within the mineral grains themselves was ignored. While the grain compressibility is small relative to the pore volume compressibility, it is more accurate to take the grain compressibility into account, which I have done in this report. Since the property that was measured was the bulk volume change under conditions of uniaxial strain and variable pore pressure, Weatherford essentially measured the coefficient C_{bp}^{uni}, and then converted it to C_{pp}^{uni} using the following conversion:

$$C_{pp}^{uni} = \frac{C_{bp}^{uni}}{\phi}, \qquad \text{(eq. 6.1)}$$

[49] Weatherford Laboratories Rock Mechanics Final Report (Uniaxial Pore Volume Compressibility Tests), Weatherford Laboratories Report WFT Labs HH-46949, WFT-MDL-00130933.

This conversion would be exact if the grains were incompressible. However, real grains are never actually incompressible, and so equation 6.1 is only an approximation. Although the mineral compressibility is small compared to the pore compressibility values, the conversion used by Weatherford would tend to slightly understate the UPVC.[50] I have accounted for the grain compressibility in my UPVC calculations.

The exact relationship between the uniaxial version of C_{bp} and the UPVC, which I have used for my calculation, is[51]

$$C_{pp}^{uni} = \frac{C_{bp}^{uni}}{\phi} + \left[\frac{2(1-2\nu)\alpha}{3(1-\nu)\phi} - 1\right]C_m, \qquad \text{(eq. 6.2)}$$

This equation will be used to convert the measured values of C_{bp}^{uni} to the desired UPVCs. In order to implement this conversion, it is necessary to calculate or estimate the porosity, ϕ, the Biot coefficient, α, the Poisson ratio, ν, and the mineral compressibility, C_m.

The effective mineral compressibility, C_m, can be calculated from the mineralogical composition of the Macondo rocks, using known handbook values of the compressibilities of the individual minerals. The Macondo samples that were examined in the uniaxial deformation tests were composed of 93% quartz, 3% plagioclase, 3% clay, and 1% feldspar. The compressibilities of these minerals are C_{quar} = 0.19 microsips, C_{plag} = 0.09 microsips, C_{clay} = 0.28 microsips, and C_{feld} = 0.18 microsips.[52] The effective compressibility of a mixture or minerals must necessarily lie between the weighted arithmetic mean of the individual compressibilities (the "Reuss average") and the weighted harmonic mean (the "Voigt average"). The mean value of the Reuss and Voigt bounds, known as the Voigt-Reuss-Hill average, is often used as the best estimate of C_m.[53] Bearing in mind that the individual mineral compressibilities are only known to two-digit accuracy, these averages are only meaningful to two digits. In units of microsips, with F representing the volume fractions of the individual minerals, these calculations are as follows:

---

[50] Specifically, the value of UPVC calculated by BP engineers using Weatherford's data (WFT-MDL-00130933) and eq. 6.1, was 6 μsips (Deposition Exhibit 8767). This is slightly lower than my estimate, which takes grain compressibility into account.

[51] Zimmerman, 2000b, eqs. 13, 26.

[52] Mavko *et al.*, 2009, p. 459.

[53] Zimmerman, 1991, pp. 16-19.

$$C_{\text{Reuss}} = F_{qrtz}C_{qrtz} + F_{plag}C_{plag} + F_{clay}C_{clay} + F_{feld}C_{feld}$$
$$= 0.93(0.19) + 0.03(0.09) + 0.03(0.28) + 0.01(0.18) = 0.190, \quad \text{(eq. 6.3)}$$

$$C_{\text{Voigt}} = 1/[(F_{qrtz}/C_{qrtz}) + (F_{plag}/C_{plag}) + (F_{clay}/C_{clay}) + (F_{feld}/C_{feld})]$$
$$= 1/[(0.93/0.19) + (0.03/0.09) + (0.03/0.28) + (0.01/0.18)] = 0.186, \quad \text{(eq. 6.4)}$$

$$C_m = (0.190 + 0.186)/2 = 0.188 \approx 0.19, \quad \text{(eq. 6.5)}$$

The conversion equation (eq. 6.2) also involves the porosity, ϕ, the Biot coefficient, α, and the Poisson ratio, ν. Poisson ratios of consolidated sandstones tend to lie between 0.1 and 0.2 (see Table 5.1). The two values of ν that were measured in the ultrasonic tests (see Section 8 and Table 8.1) were 0.13 and 0.18. Biot coefficients tend to lie between 0.6 and 0.9 (see Table 5.1), and can never exceed 1.0.[54]

So, in order *not to underestimate* the UPVCs, a maximum realistic value of the term in square brackets in equation 6.2 will be used. This maximum value can be found taking the lowest reasonable value of the Poisson ratio (ν = 0.1), the highest possible value of the Biot coefficient (α = 1.0), and the measured value of porosity (for example, ϕ = 0.217 in sample 3-6R). Using these values, it is found that the second term in equation 6.2, which was not included in Weatherford's original analysis of the data, will contribute a maximum of about 0.33 microsips to the computed value of UPVC. This value of 0.33 microsips will be used in subsequent calculations.

The uniaxial compression tests were conducted over the range of pore pressures from 11,800 psi, down to 3,800 psi. However, most of this range is not relevant to the issues at hand because, according to other BP experts the final average pore pressure after the well was eventually sealed was about 10,400 psi. Therefore, to calculate UPVC for present purposes, I will only use the data for the range of pore pressures from 11,800 psi to 10,400 psi. The UPVC values vary only slightly in the region of 10,400 psi. I will therefore base subsequent calculations on the pressure range of 11,800 to 10,400 psi. Some United States experts have offered the view that the final reservoir pressure was less than 10,400 psi. If I were to include data from pressures lower than 10,400 psi, my estimate of UPVC for the Macondo reservoir would be lower. My estimate of UPVC over the range of 11,800 psi to 10,500 psi would be nearly identical to my estimate over the 11,800 psi to 10,400 psi range.

---

[54] Ibid., p. 33.

Table 6.1 shows the relevant data and calculations for core sample 3-6R. This core sample came from a depth of 18,074.9 ft in sandstone layer M56D,[55] and had an initial porosity, at the start of the depletion test, of 21.7%. Although data was collected at pore pressure increments of roughly 1 psi, utilization of all data points would only serve to exacerbate the effects of experimental "noise", numerical round-off error, *etc*. Therefore, Table 6.1 shows only data collected at increments of roughly 100 psi. The parameter of ultimate interest is the *average* compressibility over the range of pressures that existed in the reservoir during the flow period. We can calculate this average value by averaging the individual compressibility values in Table 6.1 that were calculated for 100 psi increments, or by making a single compressibility calculation over the entire pressure range of 11,800 to 10,400 psi; both calculations would lead to the exact same result.

---

[55] Weatherford Pore Volume Compressibility Test, WFT-MDL-00082904; Post-Well Subsurface Technical Memorandum, BP-HZN-2179MDL03290054.

P_c (psi)	P_p (psi)	Axial Stress (psi)	ε_b, Bulk Strain (-)	C_{bp}(uni) $= \Delta\varepsilon_b/\Delta P_p$ (10^{-6}/psi)	C_{bp}(uni)/ϕ (10^{-6}/psi)	C_{pp}(uni) (10^{-6}/psi)
13281	11799	14794	0.000000			
13219	11699	14793	0.000071	0.84	3.87	4.20
13156	11599	14808	0.000168	1.10	5.07	5.40
13096	11500	14813	0.000290	1.52	7.00	7.33
13034	11398	14793	0.000474	1.81	8.32	8.65
12969	11299	14811	0.000653	1.82	8.40	8.73
12902	11200	14790	0.000835	1.81	8.36	8.69
12833	11100	14799	0.001014	1.75	8.06	8.39
12763	11000	14812	0.001185	1.75	8.08	8.41
12695	10901	14804	0.001363	1.72	7.90	8.23
12620	10800	14793	0.001528	1.71	7.89	8.22
12551	10700	14797	0.001707	1.77	8.13	8.46
12483	10600	14789	0.001881	1.83	8.41	8.74
12414	10500	14816	0.002072	1.87	8.61	8.94
12343	10401	14801	0.002253	1.85	8.52	8.85
12272	10300	14794	0.002442	1.79	8.25	8.58
12200	10200	14795	0.002613	1.73	7.95	8.28
12131	10100	14782	0.002787	1.73	7.97	8.30
12061	10001	14799	0.002957			

Table 6.1. Data collected during uniaxial compression test conducted on sample 3-6R, along with steps in the calculation of the UPVC.

The first column of Table 6.1 shows the lateral confining pressure. During the tests, the lateral confining pressure was decreased so as to maintain a state of zero lateral strain. The second column shows the pore pressure. The third column shows the axial (vertical) stress, which was nominally held constant at roughly 14,800 psi. The fourth column shows the total bulk strain, which is a fractional number and therefore dimensionless. These first four columns are obtained directly from the Weatherford test results.[56]

---

[56] Weatherford Pore Volume Compressibility Test, WFT-MDL-00082904.

The fifth column shows the uniaxial bulk volume compressibility, C_{bp}^{uni}, which I calculated from the ratio of the incremental bulk strain to the incremental pore pressure change. For example, the value at a pore pressure of 11,699 psi is found by using the increments that occurred over the pressure range of 11,799 psi down to 11,599 psi; mathematically, this is known as the *central difference approximation*. The sixth column shows the conversion to UPVC, using equation 6.1, as suggested by Weatherford. Finally, the last column shows the UPVC calculated using the more precise equation (eq. 6.2), with the second term taken to have the value 0.33 microsips, which was calculated above. Note that, in all cases, the values of UPVC using Weatherford's equation 6.1 are lower than those I calculate.

Table 6.2 shows the analogous data and calculations for core sample 3-16R, which came from a depth of 18,129.1 ft in sandstone layer M56E,[57] and had an initial porosity, at the start of the depletion test, of 20.6%.

---

[57] Weatherford Pore Volume Compressibility Test, WFT-MDL-00082904; Post-Well Subsurface Technical Memorandum, BP-HZN-2179MDL03290054.

P_c (psi)	P_p (psi)	Axial Stress (psi)	ε_b, Bulk Strain (-)	$C_{bp}(uni) = \Delta\varepsilon_b/\Delta P_p$ (10^{-6}/psi)	$C_{bp}(uni)/\phi$ (10^{-6}/psi)	$C_{pp}(uni)$ (10^{-6}/psi)
13286	11800	14799	0.000000			
13229	11700	14793	0.000057	0.612	2.97	3.30
13165	11599	14807	0.000123	0.645	3.13	3.46
13101	11500	14808	0.000186	0.688	3.34	3.67
13037	11400	14790	0.000260	0.765	3.71	4.04
12971	11300	14825	0.000339	0.805	3.91	4.24
12905	11200	14810	0.000421	0.824	4.00	4.33
12837	11101	14784	0.000503	0.825	4.00	4.33
12768	11000	14793	0.000586	0.840	4.08	4.41
12699	10900	14798	0.000671	0.831	4.03	4.36
12625	10799	14788	0.000753	0.815	3.96	4.29
12555	10700	14791	0.000834	0.779	3.78	4.11
12484	10600	14798	0.000908	0.819	3.98	4.31
12413	10501	14800	0.000997	0.884	4.29	4.62
12342	10401	14807	0.001084	0.836	4.06	4.39
12270	10300	14809	0.001165	0.776	3.77	4.10
12198	10200	14805	0.001240	0.760	3.69	4.02
12128	10100	14795	0.001317	0.770	3.74	4.07
12055	10000	14800	0.001394			

Table 6.2. Data collected during uniaxial compression test conducted on sample 3-16R, along with steps in the calculation of the UPVC.

Table 6.3 shows the analogous data and calculations for core sample 3-22R, which came from a depth of 18,150.0 ft in sandstone layer M56E,[58] and had an initial porosity, at the start of the depletion test, of 21.4%.

[58] Weatherford Pore Volume Compressibility Test, WFT-MDL-00082904; Post-Well Subsurface Technical Memorandum, BP-HZN-2179MDL03290054.

P_c (psi)	P_p (psi)	Axial Stress (psi)	ε_b, Bulk Strain (-)	C_{bp}(uni) $= \Delta\varepsilon_b/\Delta P_p$ (10^{-6}/psi)	C_{bp}(uni)/ϕ (10^{-6}/psi)	C_{pp}(uni) (10^{-6}/psi)
13276	11799	14792	0.000000			
13217	11700	14776	0.000061	0.727	3.40	3.73
13157	11601	14808	0.000144	0.844	3.94	4.27
13102	11501	14796	0.000229	1.00	4.67	5.00
13037	11400	14792	0.000345	1.22	5.70	6.03
12967	11301	14796	0.000473	1.28	5.96	6.26
12898	11200	14796	0.000600	1.33	6.21	6.54
12834	11100	14792	0.000740	1.32	6.18	6.51
12764	11001	14792	0.000863	1.24	5.78	6.11
12688	10901	14798	0.000986	1.24	5.79	6.12
12620	10800	14782	0.001112	1.22	5.72	6.05
12544	10700	14807	0.001232	1.22	5.70	6.03
12474	10600	14797	0.001356	1.23	5.73	6.06
12399	10501	14800	0.001476	1.23	5.73	6.06
12327	10401	14793	0.001600	1.15	5.39	5.72
12254	10300	14807	0.001708	1.09	5.11	5.44
12177	10200	14794	0.001820	1.20	5.58	5.91
12103	10100	14799	0.001947	1.16	5.42	5.75
12032	10001	14811	0.002051			

Table 6.3. Data collected during uniaxial compression test conducted on sample 3-22R, along with steps in the calculation of the UPVC.

Porous rock compressibilities should never increase during a process such as reservoir depletion, in which the stress path is such that the effective differential pressure *increases*. This is because, as the differential pressure increases, small cracks close up, and the rock becomes stiffer.[59] This is ubiquitous behavior in regard to rock compressibility. Hence, the first few values in the last column of each of these three tables, which show an *increase* in pore compressibility as the pore pressure starts to

[59] Zimmerman, 1991.

decrease, must to some extent be experimental artifacts. This phenomenon is commonly observed in compressibility tests and is attributed to the rock requiring some time to "bed itself in" to the experimental apparatus.[60]

The region of the pore compressibility values that are contaminated by experimental artifacts can easily be discerned by plotting the values as a function of pore pressure, as in Figure 6.1. For each of the three samples, the calculated UPVC increases linearly until the pore pressure declines to about 11,400 psi, after which it essentially levels off. Hence, the UPVC that is relevant for the depletion of the Macondo reservoir will be calculated from the data in the region between 11,400 psi and 10,400 psi.[61]

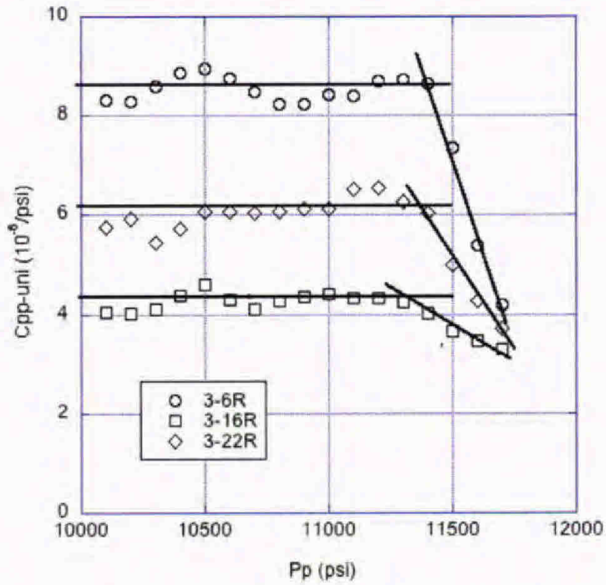


Figure 6.1. UPVC values, as calculated in Tables 6.1 through 6.3, as a function of pore pressure. For each sample, a region of spurious values that increase as the pore pressure decreases is observed until the pore pressure declines to about 11,400 psi. These values are therefore discarded when calculating the average values during depletion of the reservoir.

The average value of the uniaxial pore compressibility is thereby calculated by averaging the values in the rightmost columns of Tables 6.1 through 6.3 for pore pressures between 11,400 psi and 10,400 psi. The result is an average UPVC of 8.57 microsips for sample 3-6R, 4.34 microsips for sample 3-16R, and 6.14 microsips for sample 3-22R.

---

[60] Hoek, 1966.

[61] My estimate of UPVC would be lower were I to use the data from between 11,800 psi and 11,400 psi.

The most straightforward estimate of the overall UPVC of the reservoir is found from the mean value of these three measurements, which is 6.35 microsips. One could use a weighted average based on the thickness of the different sandstone layers, but that would only be appropriate if there were clear evidence that the sandstone layers represented different types of rock with regard to compressibility. I am not aware of any evidence of this sort with respect to the Macondo sandstone layers, so it is my opinion that an arithmetic average is the most appropriate. (However, it should be noted that my estimate of the average UPVC would be *lower* if I were to use a thickness-weighted average, since the sample that exhibited the highest compressibility, sample 3-6R, was from a thin sandstone layer.)

Finally, it may be worth noting that in the uniaxial compression tests conducted on rotary sidewall cores, the samples were loaded in a direction that corresponds to the horizontal direction relative to the samples' orientation while in the reservoir, whereas the actual deformation that occurs in the reservoir is in the vertical direction. If a rock's properties vary according to the orientation of the sample with respect to the stress, this type of behavior is known as *anisotropy*. However, appreciable mechanical (*i.e.*, compressibility) anisotropy (as opposed to anisotropy with regard to permeability) is rare in sandstones. In one of the most highly-cited papers on this topic, Thomsen[62] points out that even for rocks that are considered to be anisotropic, "in most cases of interest to geophysicists, the anisotropy is weak (10-20%)". Thomsen presents a table of measurements of mechanical anisotropy on various rocks relevant to the oil and gas industry, collated from various sources, and of the seventeen sandstones, the maximum difference between the compressibility in the vertical and horizontal direction — which occurred for a sandstone of 10% porosity — was 22%; in most cases the anisotropy was much lower. I have not seen any evidence that the Macondo sandstone is mechanically anisotropic. But even if it were, there is no reason to think that the compressibilities measured on the sidewall cores would differ from those that would have been measured on "vertical" cores, by more than a few percent. I am not aware of any scientific argument or data that supports the assumption that UPVC values measured on rotary sidewall cores should be doubled to yield the *in situ* reservoir values.

In the following two sections, this value will be compared with UPVC values obtained through somewhat more indirect methods: hydrostatic compression tests, and ultrasonic velocity tests.

---

[62] Thomsen, 1986.

7. Uniaxial Pore Volume Compressibilities Inferred from Hydrostatic "Stairstep" Porosity Measurements

Weatherford Laboratories also measured the porosity of three additional pre-incident reservoir rock samples, while the hydrostatic confining pressure was varied and holding the pore pressure constant.[63] Weatherford called these measurements the "stairstep" porosity measurements.

Since the porosity is merely the ratio of pore volume to bulk volume, changes in porosity due to stress can be expressed in terms of the changes in bulk volume and pore volume. Consequently, the coefficient that quantifies the incremental change in porosity due to an incremental change in pressure can be expressed in terms of, for example, pore compressibility, bulk compressibility, and mineral compressibility. Therefore, the hydrostatic pore volume compressibility can be determined using laboratory measurements of porosity as a function of stress. The UPVC estimates derived from Weatherford's hydrostatic porosity measurements are consistent with the estimate from the uniaxial compression tests — *i.e.*, in the range of 6 microsips or less.

Incremental changes in the applied confining pressure and/or pore pressure lead to incremental changes in porosity, according to[64]

$$d\phi = -[(1-\phi^i)C_{bc} - C_m](dP_c - dP_p) \equiv -[(1-\phi^i)C_{bc} - C_m]dP_d, \qquad \text{(eq. 7.1)}$$

where ϕ^i represents the initial porosity at the start of the process, and P_d, the differential pressure, is the difference between the confining pressure and the pore pressure. For laboratory tests, like this one, conducted at constant pore pressure and variable hydrostatic confining pressure, the change in differential pressure is equal to the change in confining pressure.

By making use of equation 5.8, equation 7.1 can be written in terms of the hydrostatic pore compressibility, C_{pp}, as follows:

$$d\phi = -\phi^i[(1-\phi^i)C_{pp} + C_m]dP_d \equiv -C_\phi dP_d, \qquad \text{(eq. 7.2)}$$

where C_ϕ is the compressibility coefficient for the porosity, and is equal to $\phi^i[(1-\phi^i)C_{pp} + C_m]$. The parameter C_ϕ is the derivative of the porosity with respect to

---

[63] Weatherford Laboratories Report WFT Labs HH-46949, WFT-MDL-00129171.

[64] Zimmerman, 1991, p. 38.

differential pressure, *i.e.*, the ratio of the incremental change in porosity divided by the incremental change in differential pressure.

Weatherford measured porosity by first increasing, and then decreasing, the confining pressure, while the pore pressure was held constant (hence the name "stairstep"). In the "loading" stage in which the confining pressure increases, the differential pressure increases, whereas in the "unloading" stage, the confining pressure decreases, and so the differential pressure decreases. During depletion of a reservoir, the stress path is more complicated, but overall, the differential pressure increases, because the pore pressure is decreasing. Therefore, the porosity data collected during the *loading* stage of these experiments are the ones that are most relevant to reservoir depletion.

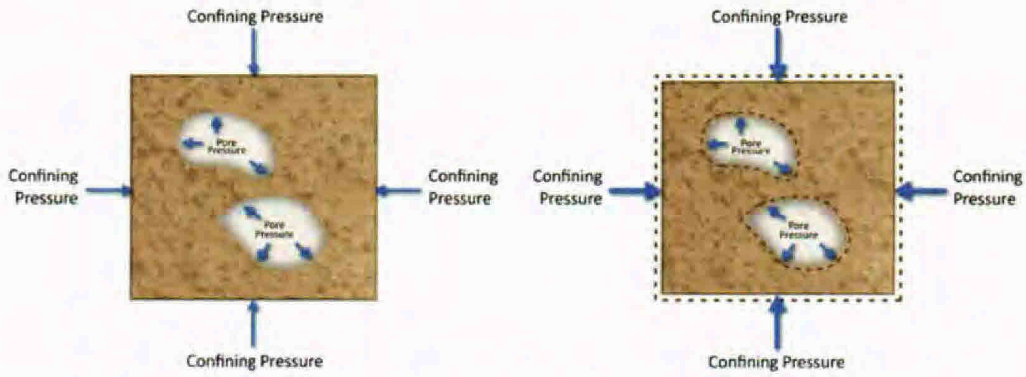


Figure 7.1. During Weatherford's hydrostatic "stairstep" test, the core samples were subjected to a confining pressure and a pore pressure (left diagram). The confining pressure was increased while the pore pressure remained constant, causing the sample to shrink (right diagram). The process was then reversed (left diagram).

Porosities were measured on three rock samples: sample 3-8R from a depth of 18,081.8 ft in sandstone layer M56D, sample 3-21R from a depth of 18.147.9 ft in sandstone layer M56E, and sample 3-25R from a depth of 18,161.0 ft in sandstone layer M56E.[65] The "initial" porosities, measured at a differential pressure of 500 psi, were 23.4%, 23.4%, and 23.7%, respectively. All three samples showed very similar behavior with regard to their variation of porosity as a function of pressure. Therefore, rather than analyze these three samples separately, and then average the resulting compressibilities, the porosities can be averaged first, from which a single compressibility can be calculated. (Any difference between the two calculations would in fact be dominated by the effects of

[65] Post-Well Subsurface Technical Memorandum, BP-HZN-2179MDL03290054.

roundoff error, as the porosities were only reported to the nearest 0.1%, and porosity changes between adjacent pressure levels on the "stairsteps" were typically on the order of a few tenths of a percent.)

The raw data, and the subsequent calculations of the hydrostatic pore volume compressibility, are shown in Table 7.1.

P_d (psi)	ϕ	ΔP_d (psi)	$\Delta\phi$	$C_\phi = -\Delta\phi / \Delta P_d$ (10^{-6}/psi)	C_{pp} (10^{-6}/psi)
500	0.235				
850		700	-0.005	7.14	39.5
1200	0.230				
1600		800	-0.002	2.50	13.3
2000	0.228				
3000		2000	-0.003	1.50	7.96
4000	0.225				
5000		2000	-0.003	1.50	7.96
6000	0.222				
7000		2000	-0.003	1.50	7.96
8000	0.219				
9000		2000	-0.002	1.00	5.31
10,000	0.217				

Table 7.1. Calculation of the hydrostatic pore volume compressibility from porosity measurements on three samples from Mississippi Canyon Blk. 252 No. 1 BP 1 Macondo Prospect.

These calculations can be explained as follows. The first two porosity measurements were made at differential pressures of 500 psi and 1200 psi (first and second columns); these values are shown in the first and third rows. The incremental change in porosity that occurred over this range of pressure was -0.005 (-0.5%) (fourth column), and the increment in differential pressure was 700 psi (third column); these values are recorded in row 2. The parameter C_ϕ is calculated, from the ratio of these two increments, to be 7.14 microsips (fifth column). This change occurred over the pressure range of 500 to 1200 psi, and so the average differential pressure in this range was 850 psi (first column), as listed in data row 2. The hydrostatic pore volume compressibility (sixth column) is then calculated as follows, based on equation 7.2:

$$C_{pp} = \frac{C_\phi - \phi^i C_m}{\phi^i (1 - \phi^i)}, \qquad \text{(eq. 7.3)}$$

where the initial porosity, ϕ^i, is the porosity at the start of the measurement, *i.e.*, ϕ^i = 0.235, and the mineral compressibility, C_m, is taken to be the value calculated in Section 6, *i.e.*, C_m = 0.19 microsips. As discussed in section 5.2, the pore compressibility decreases as the differential pressure increases — rapidly at low differential pressures, but much more gradually at higher differential pressures.

The final step in calculating the UPVC from the porosity measurements is to convert the hydrostatic pore volume compressibility values listed in the sixth column of Table 7.1, to the UPVC. As seen in equation 5.10, repeated below, this calculation requires knowledge of the Poisson ratio and the Biot coefficient:

$$C_{pp}^{uni} = C_{pp} - \frac{2(1-2\nu)\alpha}{3(1-\nu)}(C_{pp} + C_m), \qquad \text{(eq. 5.10)}$$

The Biot coefficient is, by definition, given by $\alpha = 1 - (C_m / C_{bc})$. Using equation 5.8, this can be written as

$$\alpha = \frac{\phi^i (C_{pp} + C_m)}{\phi^i (C_{pp} + C_m) + C_m}, \qquad \text{(eq. 7.4)}$$

Hence, α can be calculated from C_{pp}, using a mineral compressibility, C_m, of 0.19 microsips, and an initial porosity, ϕ^i, of 0.235.

The Poisson ratio was not reported for these samples. However, the Poisson ratios of consolidated sandstones generally lie in the range of 0.1 to 0.2, as shown in Table 5.1. More specifically, the Poisson ratios of samples 3-17R and 3-19R, as reported in Section 8, were 0.18 and 0.13, respectively. So, using equation 5.10, along with equation 7.4, to calculate α, and taking the "high" (0.2) and "low" (0.1) reasonable values of the Poisson ratio, the hydrostatic pore volume compressibilities shown in Table 7.1 can be converted to UPVCs; the relevant calculations are shown in Table 7.2.

P_d (psi)	C_{pp} (10^{-6}/psi)	α	*UPVC* (10^{-6}/psi) ($\nu = 0.1$)	*UPVC* (10^{-6}/psi) ($\nu = 0.2$)
850	39.5	0.98	16.4	20.1
1600	13.3	0.94	5.78	6.96
3000	7.96	0.91	3.56	4.25
5000	7.96	0.91	3.56	4.25
7000	7.96	0.91	3.56	4.25
9000	5.31	0.87	2.47	2.92

Table 7.2. Calculation of the UPVC from the hydrostatic pore volume compressibility (following on from Table 7.1).

The UPVC, like all porous rock compressibilities, is a function of the differential pressure. The differential pressure in this context is the difference between the mean value of the three confining stresses (the two lateral confining stresses and the axial stress), and the pore pressure. In order to directly compare the UPVC values calculated from the hydrostatic tests in Table 7.2 to those calculated from the uniaxial test in Section 6, the comparison must be made at equivalent values of the differential pressure.

At the start of the uniaxial compression tests discussed in Section 6, the differential pressure for sample 3-6R was 1986 psi (13,786 psi confining pressure minus 11,800 psi pore pressure) and increased to 2761 psi (13,161 psi confining pressure minus 10,400 psi pore pressure) when the pore pressure had declined to 10,400 psi. For sample 3-16R, the differential pressure increased from 1990 psi to 2763 psi when the pore pressure had declined to 10,400 psi. For sample 3-22R, the differential pressure increased from 1982 psi to 2748 psi when the pore pressure had declined to 10,400 psi.[66] Hence, the average differential pressure in the reservoir during the flow period was about 2372 psi. Interpolating the values in Table 7.2 to find the UPVC at a differential pressure of 2372 psi leads to a value of 4.56 microsips for an assumed Poisson ratio of $\nu = 0.1$, and a value of 5.47 microsips for an assumed Poisson ratio of $\nu = 0.2$. These values of the Poisson ratio cover the range that is expected for quartz-rich consolidated sandstones and bracket the values of 0.13 and 0.18 that were obtained using ultrasonic measurements (see Section 8). These estimated compressibilities are in reasonably good agreement with the value of 6.35 microsips that was obtained from the uniaxial compression tests. The values inferred from the hydrostatic compression tests were, in fact, slightly *lower* than those

[66] Weatherford Pore Volume Compressibility Test, WFT-MDL-00082904.

estimated from the uniaxial tests, and certainly do not support the claim that the UPVC was as large as 12 microsips.

8. Uniaxial Pore Volume Compressibility Inferred from Ultrasonic Velocities

Weatherford also performed a third test that allows estimation of UPVC: an ultrasonic velocities test. The ultrasonic wavespeeds in a dry rock depend on the elastic moduli (the bulk modulus/compressibility and the shear modulus) and the density of the rock. If the wavespeeds are measured, and the density is known, the bulk modulus and Poisson ratio can be determined, which in turn permits one to assess UPVC. Although this calculation will not provide a direct measurement of UPVC, one can use this indirect evidence as a check of the UPVCs that were estimated from the uniaxial compression tests. As shown below, the UPVC derived from Weatherford's ultrasonic measurements are roughly consistent with the 6.35-microsip UPVC estimate based on the uniaxial compression tests, but, like the other tests, are *not* consistent with values of UPVC as large as 12 microsips.

Weatherford measured ultrasonic wave velocities on two dry cores from the Macondo reservoir. These were different cores from those tested in the tests referenced in Sections 6 and 7.[67] The term "ultrasonic" refers to the fact that the wavespeeds were measured using high-frequency waves, with frequencies on the order of 10^6 Hz, or 10^6 cycles per second. Frequency dependence does not occur in a dry rock, and so the frequency of the waves is not relevant to the subsequent calculations. Wavespeeds are typically measured in feet per second. Densities are measured in grams per cubic centimeter, which can be converted to "pound-ft" units through the conversion 1 gm/cm^3 = 62.43 lbm/ft^3.[68]

The measured wavespeeds, and the inferred dynamic elastic moduli, are shown in Table 8.1, which is taken directly from the Weatherford report.[69] These wavespeeds were measured under the following stress conditions: a 2000 psi lateral confining stress; 2500 psi axial stress; and zero pore pressure. Those stresses correspond to a differential pressure of 2167 psi, which was very close to the average differential pressure of 2372 psi that the samples experienced over the relevant range of the uniaxial compression tests.

---

[67] The compressional wave velocity was actually measured on three cores, but the shear wave velocity was measured on only two of those cores. Since both wavespeeds are needed in order to calculate the bulk modulus, only the two cores for which both wavespeeds are available will be used in the following analysis.

[68] Mavko *et al.*, 2009, p. 454.

[69] Rock Mechanics Final Report (Acoustic Velocities / Mohr-Coulomb Failure Analysis), Weatherford Laboratories Report WFT Labs HH-46949, WFT-MDL-00082902.

Sample No.	Depth (ft)	Confining Pressure (psi)	Bulk Density (gm/cm^3)	V_p (ft/sec)	V_s (ft/sec)	Bulk Modulus, K (10^6 psi)	Shear Modulus, G (10^6 psi)	Poisson Ratio, ν
3-17R	18131.90	2000	2.04	10481	6517	1.46	1.17	0.18
3-19R	18141.90	2000	2.00	10551	6861	1.31	1.27	0.13

Table 8.1. Dynamic properties of two dry cores from the Macondo reservoir; data taken from Weatherford's test results.[70]

The calculations that are required to derive the elastic moduli, K and G, from the wavespeeds, V_p and V_s, can be explained as follows, using as an example the shear modulus calculated for sample 3-17R. According to equation 5.12, the shear modulus is given by $G = \rho V_s^2$. However, care must be taken to properly convert the units. First, the density in grams per cubic centimeter (gm/cm^3) must be converted to pound mass per cubic foot (lbm/ft^3), according to the conversion 1 gm/cm^3 = 62.43 lbm/ft^3,[71] where lbm denotes "pound mass". All terms involving feet, such as are used for the wavespeeds, must be converted to inches, as are used for the shear modulus. Furthermore, the "pound mass" units that are used for the density must be converted to "pound force" units that are used in the elastic moduli, according to the conversion 1 lbf = 32.174 lbm•ft/s^2. So:

$$G = \rho V_s^2$$

$$= \left(2.04\frac{\text{gm}}{\text{cm}^3}\right)\left(62.43\frac{\text{lbm/ft}^3}{\text{gm/cm}^3}\right)\left(6517\frac{\text{ft}}{\text{sec}}\right)^2\left(\frac{1\ \text{lbf}}{32.174\ \text{lbm ft/s}^2}\right)\left(\frac{1\ \text{ft}^2}{144\ \text{in}^2}\right) \qquad \text{(eq. 8.1)}$$

$$= 1.17 \times 10^6\ \text{psi}.$$

Similarly, using the wavespeeds and densities reported in Table 8.1, along with equation 5.13, the bulk moduli of the two samples, 3-17R and 3-19R, can be calculated to be 1.46×10^6 psi and 1.31×10^6 psi, respectively.

---

[70] Rock Mechanics Final Report (Acoustic Velocities / Mohr-Coulomb Failure Analysis), Weatherford Laboratories Report WFT Labs HH-46949, WFT-MDL-00082902.

[71] Mavko *et al.*, 2009, p. 454.

The next step is to convert the bulk moduli to the hydrostatic pore volume compressibility, C_{pp}. To accomplish this, the bulk modulus, K, is first converted to the bulk compressibility, C_{bc}, through the definition $C_{bc} = 1/K$. Next, the hydrostatic pore volume compressibility is calculated from equation 5.8, which states that $C_{bc} = C_m + \phi(C_{pp} + C_m)$, *i.e.*,

$$C_{pp} = \frac{C_{bc} - C_m(1+\phi)}{\phi}, \qquad \text{(eq. 8.2)}$$

The calculation prescribed in equation 8.2 requires knowledge of the mineral compressibility, C_m, and the porosity, ϕ. As explained in Section 6, the mineral compressibility is 0.19 microsips.

The porosity of each sample can be calculated from knowledge of its bulk density and its mineral density through the relationship $\rho_b = \rho_m(1-\phi)$. Using the average mineral density reported by Weatherford of 2.65 gm/cm^3 the porosity can be found from[72]

$$\phi = 1 - (\rho_b / \rho_m), \qquad \text{(eq. 8.3)}$$

The calculated porosities are shown in Table 8.2.

Using the calculated porosities, and the mineral compressibility of 0.19 microsips, the hydrostatic pore volume compressibilities of these two samples can be calculated from equation 2.2 and are shown in Table 8.2.

Finally, using equation 5.10, one can use the hydrostatic pore volume compressibility to calculate the UPVC. This calculation requires knowledge of the Biot coefficient, α, which is defined by $\alpha = 1 - (C_m/C_{bc})$.[73] The calculated UPVCs are shown in the last column of Table 8.2.

---

[72] Weatherford Laboratories Report WFT Labs HH-46949. WFT-MDL-00129171.

[73] Zimmerman, 1991, p. 33; Mavko *et al.*, 2009, p. 45.

Sample No.	Bulk Density (gm/cm³)	Porosity	Bulk Comp. C_{bc} (10^{-6}/psi)	Biot coefficient, α	Poisson Ratio, ν	Hydrostatic Pore Compressibility, C_{pp} (10^{-6}/psi)	Uniaxial Pore Compressibility, *UPVC* (10^{-6}/psi)
3-17R	2.04	0.230	0.685	0.722	0.18	1.96	1.15
3-19R	2.00	0.245	0.763	0.751	0.13	2.15	1.15

Table 8.2. Pore compressibilities of two cores from the Macondo reservoir, as calculated from the ultrasonic velocities shown in Table 8.1.

These uniaxial pore compressibilities, which have been estimated from the ultrasonic data, both have been calculated as 1.15 microsips, which is much lower than the values estimated from the static uniaxial measurements (Section 6), or from the static hydrostatic measurements (Section 7). It is in fact well known that dynamic compressibilities are usually lower than static compressibilities.[74] This phenomenon is usually attributed to the effects of frictional forces acting along adjacent grains. During the small strains that occur during seismic or ultrasonic wave propagation, the small motions are not sufficient to overcome the frictional forces, and the grains are "locked" together, leading to stiffer behavior. On the other hand, during the large strains that occur during static deformations, frictional forces are overcome, and the rock behaves in a less stiff manner.

Although there are no simple relations between dynamic and static elastic moduli, there are some correlations, as reviewed by Mavko *et al.*[75] According to the correlation developed by Wang and Nur,[76] rocks having dynamic elastic moduli in the range shown in Table 8.1 will tend to have static moduli that are about three times larger than the dynamic values. Use of this factor of about 3 to convert "dynamic" UPVC to "static" UPVC, would have led to static values of UPVC that are in the range of 4 microsips — a value that, while certainly not precise, is roughly consistent with a value of UPVC in the range of about 6 microsips, as was calculated from the uniaxial compression tests. This analysis does not support UPVC as large as 12 microsips.

---

[74] Mavko *et al.*, 2009, pp. 76-80.

[75] Mavko *et al.*, 2009.

[76] Wang and Nur, 2000.

9. Comparison with Values and Correlations from the Literature

I also have compared my 6.35-microsip average UPVC estimate (based on the uniaxial compression test data) with previously reported UPVC values for consolidated sandstones. That literature shows that my value is actually at the upper range reported UPVC values for sandstones having porosities in the range of 20% to 23% — the general level of porosity of the Macondo reservoir.

For example, the classic correlation developed by Hall[77] indicates that consolidated sandstones of 20-23% porosity will have hydrostatic pore compressibilities of about 3 to 4 microsips. Hall used a primitive experimental set-up in which the cores were placed in Lucite holders (rather than thin, flexible sheathing as is now the practice), so it is difficult to quantify the extent to which the nominal confining stress of 3000 psi was transmitted through the Lucite to the sample. Hall depleted the cores from an initial pore pressure of 1500 psi down to a final pore pressure of 200 psi. Assuming that the actual confining stress acting on the cores was 3000 psi, then the mean differential pressure during Hall's experiments would have been [(3000-1500)+(3000-2000)]/2 = 2150 psi, which is close to the mean differential pressure that was acting on the Macondo cores during the uniaxial depletion experiments discussed in Section 6.

Hall found that, when comparing the compressibilities of different rocks, the pore compressibility decreases as the nominal porosity increases. For rocks with initial porosities of 10% or greater, Hall found hydrostatic pore compressibilities of less than 6 microsips, which would translate to UPVCs of less than 4 microsips. For an initial porosity of 20 to 23%, Hall's data would imply an even lower hydrostatic pore volume compressibility of 3 to 4 microsips. (The *uniaxial* pore volume compressibility would be approximately 1.5 to 2 microsips.) Although Hall's sample size is small, it is yet another data point showing more consistency with a 6-microsip UPVC for Macondo than with a UPVC of 12 microsips.

Another well-known data set was collected by Newman,[78] who tested a larger number of samples — 197 sandstone samples from twenty-nine different reservoirs. Newman divided his samples into three categories:

---

[77] Hall, 1953.

[78] Newman, 1973.

1. *Consolidated* samples, defined as "hard" rocks for which thin edges could not be broken off the samples by hand;

2. *Friable* samples, defined as samples that could be cut into cylinders, but for which the edges of the samples could be broken off by hand;

3. *Unconsolidated* samples, defined as samples that would fall apart under their own weight unless they had undergone special treatment such as freezing.

The Macondo cores, which had been buried at subsurface depths of greater than 13,000 feet, and at confining pressures of more than 10,000 psi, would be expected to fall into Newman's category of "consolidated" sandstones. This is consistent with the "unconfined compressive strengths" of 863 psi, 1645 psi, and 2405 psi that were measured by Weatherford on three Macondo samples,[79] which indicate that the Macondo rocks were very well consolidated.

Newman's experimental procedure and data analysis protocol are not entirely clear from the description given in his paper. In particular, it seems that in some cases he measured C_{pp}, and in other cases measured C_{pc} (as defined in Section 5), and most likely did not distinguish between these two parameters. However, these two compressibility coefficients differ numerically only by the mineral compressibility, C_m, which for the Macondo cores is on the order of 0.19 microsips. Thus, the distinction between C_{pp} and C_{pc} is of no importance to the current discussion, which is only semi-quantitative.

In Newman's data set of nearly one hundred consolidated sandstones, the fifty-five samples with porosities greater than 10% each had a reported hydrostatic pore volume compressibility of less than 11 microsips, which would correspond to UPVC values of less than 7 microsips. The twelve samples with porosities exceeding 20% each had reported hydrostatic pore volume compressibilities of less than 5 microsips.

The UPVC value of 6.35 microsips that has been estimated in this report for the Macondo reservoir is therefore not unexpectedly low, and does not seem to require any special explanation. This value is in fact at the upper edge of the values that can be found in some classic data collections. On the other hand, a UPVC of 12 microsips for

---

[79] Rock Mechanics Final Report (Acoustic Velocities / Mohr-Coulomb Failure Analysis), Weatherford Laboratories Report WFT Labs HH-46949, WFT-MDL-00082902.

consolidated sandstones having porosities of about 20% would indeed be anomalous, and inconsistent with most values found in the petroleum engineering literature.

10. Conclusion

The opinions that I have presented in this report are based on established techniques of data analysis in the field of rock mechanics, and the results are supported by three independent laboratory measurements on eight different samples and a review of the literature. The best estimate of UPVC for the Macondo reservoir — based on all the laboratory data — is no more than 6.35 microsips, and there is no support in the data for a UPVC of 12 microsips.

My opinions rest on analysis of test data from eight rotary sidewall cores, each of which yields a UPVC estimate of around 6 microsips or less. The chances of picking eight samples at random such that each sample somehow had a compressibility that was *lower* than the actual average compressibility are $1/2^8$ — or 0.39%. This gives us a high degree of confidence that the measurements discussed in this report are unlikely to significantly understate the UPVC of the sandstone in the Macondo well. While the available rock samples represent only a small fraction of the reservoir, the data from testing those cores represents the best data we have available, and an entire field of petroleum engineering is based on relying on information obtained from core sample analysis.[80]

I am receiving £280 per hour for my work on this case, although I understand that the contracting agency of my university, by whom I am paid, receives £350 per hour.

Robert W. Zimmerman

[80] I may form additional opinions after reviewing expert reports received after this report is submitted.

Appendix 1
References

Biot, M. A. (1941) General theory of three-dimensional consolidation. *Journal of Applied Physics*, **12**, 155-164.

Carpenter, C. B., and Spencer, G. B. (1940) Measurement of compressibility of consolidated oil-bearing sandstones, *U. S. Bureau of Mines Report 3540*, 20 pp.

Crawford, B. R., Sanz, P. F., Alramahi, B., and DeDontney, N. L. (2011) Modeling and prediction of formation compressibility and compactive pore collapse in siliciclastic reservoir rocks, in *Proceedings of the 45th U.S. Rock Mechanics Symposium*, San Francisco, 26-29 June 2011, paper ARMA 11-384.

Dake, L. P. (1978) *Fundamentals of Reservoir Engineering* (Developments in Petroleum Science, Vol. 8), Elsevier, Amsterdam, 443 pp.

Detournay, E., and Cheng, A. H.-D. (1993) Fundamentals of poroelasticity. In: *Comprehensive Rock Engineering*, J. A. Hudson, ed., Pergamon, Oxford, pp. 113-171.

Fjær, E., Holt, R. M., Raaen, A. M., Risnes, R., and Horsrud, P. (2008) *Petroleum Related Rock Mechanics*, 2nd ed., (Developments in Petroleum Science, Vol. 53), Elsevier, Amsterdam, 514 pp.

Fortin, J., Guéguen, Y., and Schubnel, A. (2007) Effects of pore collapse and grain crushing on ultrasonic velocities and Vp/Vs. *Journal of Geophysical Research*, **112**, B08207.

Geertsma, J. (1957) The effect of fluid decline on volumetric changes of porous rocks. *Petroleum Transactions of the AIME*, **210**, 331-340.

Geertsma, J. (1973) Land subsidence above compacting oil and gas reservoirs. *Journal of Petroleum Technology*, **25**, 734-744.

Hall, H. N. (1953). Compressibility of reservoir rocks. *Petroleum Transactions of the AIME*, **198**, 309-311.

Hoek, E. (1966) Rock Mechanics - an introduction for the practical engineer. Part II. Laboratory techniques in rock mechanics. *Mining Magazine*, June 1966.

Jaeger, J. C., Cook, N. G. W., and Zimmerman, R. W. (2007) *Fundamentals of Rock Mechanics*, 4th ed., Blackwell, Oxford, 475 pp.

Matthews, C. S., and Russell, D. G. (1967) *Pressure Buildup and Flow Tests in Wells* (SPE Monograph Volume 1), Society of Petroleum Engineers, Dallas, 172 pp.

Mavko, G., Mukerji, T., and Dvorkin, J. (2009) *The Rock Physics Handbook*, 2^{nd} ed., Cambridge University Press, New York, 511 pp.

Newman, G. H. (1973) Pore-volume compressibility of consolidated, friable, and unconsolidated reservoir rocks under hydrostatic loading. *Journal of Petroleum Technology*, **25**, 129-134.

Thomson, L. Weak elastic anisotropy. *Geophysics*, **51**, 1954-1966.

Walker, G. (2010) *Weatherford Laboratories - X-Ray Diffraction - Native* (BP-HZN-2179MDL02394187), April 24, 2010.

Wang, Z., and Nur, A., eds. (2000) *Seismic and Acoustic Velocities in Reservoir Rocks, vol. 3*, Society of Exploration Geophysicists, Tulsa, Ok.

Zimmerman, R. W. (1991) *Compressibility of Sandstones* (Developments in Petroleum Science, Vol. 29), Elsevier, Amsterdam, 173 pp.

Zimmerman, R. W. (2000a) Coupling in poroelasticity and thermoelasticity. *International Journal of Rock Mechanics and Mining Sciences*, **37**, 79-87.

Zimmerman, R. W. (2000b) Implications of static poroelasticity for reservoir compaction, in *Proceedings of the 4th North American Rock Mechanics Symposium*, A. A. Balkema, Rotterdam, pp. 169-172.

Zimmerman, R. W. (2000c) Pore compressibility under uniaxial strain, in *Proc. 6th International Symposium on Land Subsidence*, National Research Council of Italy, Venice, pp. 57-65.

Zimmerman, R. W., Somerton, W. H., and King, M. S. (1986) Compressibility of porous rocks, *Journal of Geophysical Research*, **91**, 12765-12778.

Appendix 2
Curriculum Vitae

Prof. Robert W. Zimmerman
Department of Earth Science and Engineering
Imperial College
London SW7 2AZ
United Kingdom
tel: 44-020-7594-7412
fax: 44-020-7594-7444
email: r.w.zimmerman@imperial.ac.uk

Birth

9 July 1956; New York, N.Y.

Education

UNIVERSITY OF CALIFORNIA, Berkeley, Calif.

Ph.D. in Mechanical Engineering, June 1984; GPA: 3.87/4.00

Major Field: Solid Mechanics

Minor Fields: Applied Mathematics, Thermodynamics/Heat Transfer

COLUMBIA UNIVERSITY, New York, N.Y.

M.S. in Mechanical Engineering, May 1979; GPA: 3.93/4.00

B.S. in Mechanical Engineering, May 1977; GPA: 3.77/4.00

BRONX HIGH SCHOOL OF SCIENCE, New York, N.Y.

Career

IMPERIAL COLLEGE, London (January 2008 – present): Professor of Rock Mechanics. Lecturing in geodynamics and rock physics. Research in rock mechanics, fractured rock hydrology, and petrophysics, with applications to mining, nuclear waste disposal, carbon sequestration, and petroleum engineering.

ROYAL INSTITUTE OF TECHNOLOGY, Stockholm (September 2004 – December 2007): Professor of Engineering Geology, Head of Division of Engineering Geology and Geophysics. Lecturing in rock mechanics. Research in rock mechanics, fractured rock hydrogeology, and petrophysics, with applications to nuclear waste disposal and petroleum engineering.

IMPERIAL COLLEGE, London (September 1995 – September 2004): Reader in Rock Mechanics, 1999-2004; Governors' Lecturer in Rock Mechanics and Petroleum Engineering, 1995-99. Lecturing in rock mechanics, soil mechanics, engineering mechanics, flow through porous media, and applied mathematics. Research in rock mechanics, fractured rock hydrology, petroleum reservoir engineering, and petrophysics.

LAWRENCE BERKELEY NATIONAL LABORATORY, Berkeley, Calif. (October 1987 – September 1995; part-time, 1995–1999): Staff Scientist, Earth Sciences Division. Mathematical modelling of hydrological and geomechanical processes associated with underground radioactive waste isolation and geothermal energy production. Basic research on physical properties of geological media.

UNIVERSITY OF CALIFORNIA, Berkeley, Calif. (September 1986–June 1988): Lecturer. Undergraduate and postgraduate courses in rock mechanics, porous media flow, heat transfer, fluid mechanics, and applied mathematics.

UNIVERSITY OF CALIFORNIA, Berkeley, Calif. (July 1984–June 1986): Post-doctoral Researcher. Research on the effect of microstructure on the physical properties of porous and heterogeneous geological materials.

Editorships, Committees, etc.

Editor-in-Chief, International Journal of Rock Mechanics, 2006–present

Editorial Board, International Journal of Rock Mechanics, 2001–2005

Editorial Board, *Transport in Porous Media*, 1996–present

Editorial Board, International Journal of Engineering Science, 2007–present

Member, Committee on Poromechanics, Amer. Soc. Civil Eng., 2004–present

Member, Commission on Petroleum Geomechanics, ISRM, 2009-present

Fellowships and Awards

MTS Visiting Professor of Geomechanics, Department of Civil Engineering, University of Minnesota (April 2011)

Maurice A. Biot Medal for Poromechanics from the American Society of Civil Engineers (2010)

Best paper in *Magazine of Concrete Research* from the Institute of Civil Engineers (2009)

Award for Excellence in Reviewing from the American Geophysical Union (2005)

Imperial College Award for Excellence in Teaching (1997)

SOHIO (BP) Post-doctoral Fellowship (1985–86)

Mobil Foundation Fellowship (1982–83)

U.S. Dept. of Energy Mining and Mineral Resources Fellowship (1981–82)

Affiliations

American Geophysical Union (AGU)

American Society of Mechanical Engineers (ASME)

American Society of Civil Engineers (ASCE)

Society of Petroleum Engineers (SPE)

Tau Beta Pi

Pi Tau Sigma

Consulting

CALIFORNIA BUREAU OF STATE LANDS (January 1986–December 1986): Consultant and expert witness in Long Beach Oilfield Reservoir Properties Determination and Equity Arbitration.

INTERPORE, Inc., Irvine, Calif. (January 1986–December 1987): Stress and deformation of dental implants.

TERRA TEK, Inc., Salt Lake City, Utah (May 1991 – December 1992): Consultant on modelling and measurement of physical properties of petroleum reservoir rocks.

DOVE ENERGY, Aberdeen, UK (Sept 2009 – October 2010): Consultant on rock properties in various Middle East petroleum reservoirs.

EATEC, Ltd., Bristol, UK (July 2012 – ongoing): Consulting on numerical modelling of rock-bit interactions.

Invited Talks

"Lubrication approximation for fluid flow in rock fractures", *2nd International Conference on Industrial & Applied Mathematics*, Washington, D.C., 11 July 1991.

"Effective hydraulic conductivity of 2-D porous media containing elliptical inhomogeneities", *Fall Annual Meeting American Geophysical Union*, San Francisco, 9 December 1992.

"Predicting the elastic moduli and conductivity of heterogeneous materials using the differential effective medium theory", *Dept. Mater. Sci. & Mech. / Dept. of Physics & Astronomy, Michigan State University*, Lansing, Michigan, 17 November 1993.

"Unsaturated flow in fractured rocks", invited keynote talk, *AGU Chapman Conf. on Aqueous-Phase and Multiphase Transport in Fractured Rock*, Burlington, Vermont, 14 September 1994.

"Effective stress laws for the mechanical and transport properties of porous rocks", *Prager Symposium on Heterogeneous Solids, 31st Annual Meeting Society of Engineering Science*, College Station, Texas, 10 October 1994.

"Hydraulic conductivity of rock fractures", *Society of Petroleum Engineers Los Angeles Chapter*, Los Angeles, 18 October 1994.

"Fluid flow through rock fractures: beyond the cubic law", *Department of Earth Sciences, Oxford University*, Oxford, 10 May 1996.

"Fluid flow through rock fractures: beyond the cubic law", *London Petrophysical Society*, London, 20 May 1996.

"Crack aspect ratio distributions and seismic velocities in rocks", *Schlumberger Cambridge Research Laboratory*, Cambridge, 23 May 1996.

"Fluid flow through rock fractures: beyond the cubic law", *Department of Applied Mathematics and Theoretical Physics, Cambridge University*, Cambridge, 14 November 1996.

"Navier-Stokes simulations of fluid flow in rock fractures", *Nottingham Geomechanics Centre, University of Nottingham*, Nottinghham, 6 March 2003.

"Predicting the permeability of sedimentary rocks from image analysis of the pore space, *SINTEF Research Centre,* Trondheim, Norway, 25 September 2003.

"Fluid flow in rock fractures", *Fall Annual Meeting American Geophysical Union*, San Francisco, 12 December 2003.

"Fluid flow in rock fractures and fractured rock masses", *Department of Earth Sciences, ETH,* Zurich, 27 January 2005.

"On the relationship between poroelasticity and thermoelasticity", *SIAM Conference on Mathematical and Computational Issues in Geosciences,* Avignon, France, 8 June 2005.

"Fluid flow in rock fractures", invited keynote talk, *11th International Conference Computer Methods Applied to Geomechanics*, Torino, Italy, 23 June 2005.

"Pore-scale analysis of flow in sedimentary rocks, and predictions of permeability from 2-D images", *BP Institute, Cambridge University*, Cambridge, 13 October 2006.

"Fluid flow in rock fractures", and "Fundamentals of poroelasticity", invited lecture tour of eight Chinese universities, sponsored by the *Chinese Academy of Sciences and the International Society for Rock Mechanics*, 22 October – 4 November 2006.

"Estimating the elastic moduli and permeability of porous rocks using two-dimensional pore space images", *SIAM Conference on Mathematical and Computational Issues in Geosciences,* Santa Fe, New Mexico, 19 March 2007.

"A simple model for the variation of the transmissivity of a rock fracture under normal stress", *Fall Annual Meeting American Geophysical Union*, San Francisco, 10 December 2007.

"Relationship between permeability, elastic moduli and pore structure in porous geological media", *Fall Annual Meeting American Geophysical Union*, San Francisco, 11 December 2007.

"Some rock mechanics issues in petroleum engineering", invited keynote talk, *European Rock Mechanics Symposium (Eurock 2010),* Lausanne, Switzerland, 15-18 June 2010.

"Elastic moduli of solids containing spheroidal pores", Warren Lecture, *Department of Civil Engineering, University of Minnesota*, Minneapolis, 29 April 2011.

"Some new developments in modeling failure, fracture and fragmentation of rocks", invited keynote talk, *7th Asian Rock Mechanics Symposium (ARMS 2012),* Seoul, Korea, 15 October 2012.

"The history and role of the cubic law for fluid flow in fractured rocks", *Fall Annual Meeting American Geophysical Union*, San Francisco, 3 December 2012.

Conference Sessions Chaired

Chaired session on "Elasticity", *ASME Applied Mechanics Conference*, Berkeley, Calif., 20-22 June 1988.

Chaired session on "Mathematical and Computational Methods in Fluid Mechanics", *2nd International Congress Industrial & Applied Mathematics*, Washington, D.C., 8-12 July 1991.

Chaired session on "Physical Properties of Porous Rocks", *Fall Annual Meeting American Geophysical Union*, San Francisco, Calif., 14-18 December 1996.

Chaired session on "Functionally Graded Materials", *International Symposium Thermal Stresses*, Rochester, N.Y., 8-11 June 1997.

Chaired session on "Remediation in Fractured Rock", *International Conference Fractured Rock*, Toronto, 26-28 March 2001.

Chaired session on "Fractures and Fluid Flow", *38th U.S. Rock Mechanics Symposium*, Washington, D.C., 7-10 July 2001.

Chaired session on "Brittle Deformation and Fracture Processes", *4th Euroconference Rock Physics & Geomechanics*, Kijkduin, Netherlands, 8-12 September 2003.

Chaired session on "Recent Advances in Fracture Flow Modelling", *2nd Int. Symp. Dynamics of Fluids in Fractured Rock*, Berkeley, Calif., 10-12 February 2004.

Chaired session on "Fracture Mechanics of Rock", *SINOROCK-2004*, Yichang, China, 18-21 May 2004.

Chaired session on "Fluid Transport in Rocks", *6th Euroconference Rock Physics & Geomechanics*, Potsdam, Germany, 20-23 September 2004.

Chaired session on "CO_2 sequestration", *6th Euroconference Rock Physics & Geomechanics*, Oleron, France, 18-22 September 2005.

Chaired session on "Rock Physics", *Fall Annual Meeting American Geophysical Union*, San Francisco, Calif., 11-15 December 2006.

Chaired session on "Coupled Processes" at the *2nd Canada-US Rock Mechanics Symposium*, San Francisco, Calif., 29 June-02 July 2008.

Chaired session on "Effective Medium Theory" at the *1st Southern Hemisphere Int. Rock Mechanics Symposium, Perth, Australia,* 15-19 September 2008.

Chaired session on "Rock Mass Characterization", at the *4nd US-Canada Rock Mechanics Symposium*, Asheville, North Carolina, 28 June-01 July 2009.

Chaired session on "Underground Testing, Monitoring, and Modeling in Different Formations", *Fall Annual Meeting American Geophysical Union,* San Francisco, Calif., 3-5 December 2012.

Conferences and Special Sessions Organised

1. Organised Special Session "Fractures and Fluid Flow", *38th Rock Mechanics Symposium,* Washington, D.C., 7-10 July 2001.

2. Scientific organising committee, *5th Euroconference on Rock Physics and Geomechanics,* Kijkduin, Netherlands, 8-12 September 2003.

3. Scientific organising committee, *5th Euroconference on Rock Physics and Geomechanics,* Potsdam, Germany, 20-23 September 2004.

4. Scientific advisory committee, *3rd Maurice A. Conference on Poromechanics,* Norman, Oklahoma, 24-27 May 2005.

5. International advisory committee, 11th International Conference on Computer Methods Applied to Geomechanics, Torino, Italy, 19-24 June 2005.

6. Scientific organising committee, *6th Euroconference on Rock Physics and Geomechanics,* Oleron, France, 18-22 September 2005.

7. International advisory committee, *1st Southern Hemisphere International Rock Mechanics Symposium,* Perth, Australia, 15-19 September 2008.

8. International advisory committee, 12th International Conference on Computer Methods Applied to Geomechanics, Goa, India, 1-6 October 2008.

9. Scientific advisory committee, *4th Maurice A. Biot Conference on Poromechanics,* New York, 8-10 June 2009.

10. Scientific organising committee, *9th Euroconference on Rock Physics and Geomechanics,* Trondheim, Norway, 17-21 October 2011.

11. International advisory committee, *3rd Sinorock Conference*, Shanghai, China, 18-20 June 2013.

External Examiner of Ph.D. Theses

1. *Geometry, Mechanics and Transmissivity of Rock Fractures*, Flavio Lanaro, Dept. of Civil Engineering, Royal Institute of Technology, Stockholm (supervisor: Prof. Ove Stephansson), April 2001.

2. *An Investigation into the Effect of Pore Structure on Gas/Liquid Permeability Corrections in Crystalline Rocks*, Matthew Phillips, Dept. of Geological Sciences, University College, London (supervisor: Prof. John Barker), February 2002.

3. *The Simulation of Single-Phase, Compressible Fluid Flow in Fractured Petroleum Reservoirs using Finite Elements*, Shane Hattingh, Faculty of Science, University of Cape Town, South Africa (supervisor: Prof. B. D. Reddy), March 2002.

4. *Stochastic Reservoir Characterization and Data Integration Near Wells,* Jo Eidsvik, Department of Mathematical Sciences, Norwegian University of Science and Technology (NTNU), Trondheim (supervisor: Prof. Henning Omre), September 2003.

5. *Rock Physics of Extensional Faults and their Seismic Imaging Properties*, Lill-Tove Wetjen Sigernes, Department of Petroleum Engineering and Applied Geophysics, Norwegian University of Science and Technology (NTNU), Trondheim (supervisor: Prof. Egil Tjaland), September 2004.

6. *A Study on New Approaches for Delineating Groundwater Protection Zones in Fractured-Rock Aquifers*, Júlio Ferreira Carneiro, Dept. of Earth Sciences, University College, London (supervisor: Prof. John Barker) January 2005.

7. *Upscaling of Flow, Transport and Stress Effects in Fractured Rocks,* Johan Öhman, Department of Earth Sciences, Uppsala University, Uppsala, Sweden (supervisor: Prof. Auli Niemi) April 2005.

8. *Experimental and Modeling Studies on the Spreading of Non-Aqueous Phase Liquids in Heterogeneous Media*, Fritjof Fagerlund, Department of Earth Sciences, Uppsala University, Uppsala, Sweden (supervisor: Prof. Auli Niemi) 19 January 2007.

9. *Thermo-Poro-Mechanical Behavior of a Hardened Oil-well Cement Paste*, Siavesh Ghabezloo, Ecole Nationale des Ponts et Chaussées, Paris, France (supervisor: Prof. Jean Sulem) 26 September 2008.

10. *Subsurface Impact of CO2: Response of Carbonate Rocks and Wellbore Cement to Supercritical CO2 Injection and Long-term Storage*, Emilia Liteanu, Department of Earth Sciences, Utrecht University, Utrecht, Netherlands (supervisor: Prof. Chris Spiers) 30 November 2009.

11. *The Impact of Steam Injection on Fracture Permeability in Carbonate Reservoirs,* Ruqaiya Al-Zadjali, School of Earth and Environment, Leeds University, Leeds, UK (supervisor: Prof. Quentin Fisher) 19 May 2011.

Ph.D. Theses Supervised

1. Di-Wen Chen, *Coupled Stiffness-Permeability Analysis of a Single Rough-Surfaced Fracture by the Three-Dimensional Boundary Element Method*, (co-supervisor: Neville Cook), UC Berkeley, May 1990.

2. Erika Schlueter, *Predicting the Transport Properties of Sedimentary Rocks from Microstructure*, (co-supervisors: Neville Cook, Paul Witherspoon), UC Berkeley, May 1995.

3. Melanie Lutz, *Elastic and Thermoelastic Behavior of Materials with Continuously-Varying Elastic Moduli*, (co-supervisor: Paulo Monteiro), UC Berkeley, August 1995.

4. In-Wook Yeo, *Anisotropic Hydraulic Properties of a Rock Fracture Under Normal and Shear Loading*, (co-supervisor: Michael de Freitas), Imperial College, December 1997. Examiner: John Barker, University College London.

5. Hamed Al-Sharji, *Experimental Observation and Measurement of the Flow of Oil and Water through Polymer Gels*, (co-supervisor: Carlos Grattoni), Imperial College, November 2000.

6. Carlos Romero, *A Genetic Algorithm for Reservoir Characterisation using Production Data*, (co-supervisor: Jonathan Carter), Imperial College, November 2000.

7. Sourith Sisavath, *Fundamental Pore-Scale Modelling of Single-Phase Flow through Sedimentary Rocks*, (co-supervisor: Xudong Jing), Imperial College, December 2000.

8. Canghu Yang, *Mathematical Modelling the Flow of Water and Oil through Polymer Gels*, (co-supervisor: Ann Muggeridge), Imperial College, March 2001.

9. Peter Lock, *Estimating the Permeability of Reservoir Sandstones using Image Analysis of Pore Structure*, (co-supervisor: Xudong Jing), Imperial College, November 2001. Examiner: Ida Fabricius, Technical University of Denmark.

10. Widad Al-Wardy, *Analytical and Experimental Study of the Poroelastic Behaviour of Clean and Clay-Rich Sandstones*, Imperial College, July 2003. Examiner: Ian Main, University of Edinburgh.

11. Rifaat Al-Mjeni, *The Effect of Clay, Salinity and Saturation on the High-Frequency Electrical Properties of Shaly Sandstones*, (co-supervisor: Xudong Jing), Imperial College, July 2003.

12. Azzan Al-Yaarubi, *Numerical and Experimental Study of Fluid Flow in a Rough-Walled Rock Fracture,* Imperial College, August 2003. Examiner: Axel Makurat, Shell SIEP.

13. John Matthews, *Geological Controls on the Transition Zone in Hydrocarbon Reservoirs*, (co-supervisor: Jonathan Carter), Imperial College, June 2004.

14. Sultan Al-Mahrooqi, *Investigation of Wettability of Sandstone Cores using NMR Relaxation Times*, (co-supervisor: Ann Muggeridge), Imperial College, December 2004.

15. Robert Seymour, *Effect of Stress on Seismic Velocities in Reservoir Rocks,* Imperial College, March 2005. Examiner: Patrick Corbett, Heriot-Watt University.

16. Adel Al-Ajmi, *Analysis of Wellbore Stability using a True-Triaxial Failure Criterion*, Royal Institute of Technology, June 2006. Opponent: Bernt Aadnoy, University of Stavanger.

17. Mathieu Jurgawczynski, *Predicting Petrophysical Properties of Carbonate Rocks from Two-Dimensional Images*, Imperial College, February 2007. Examiner: Ole Torsaeter, NTNU Trondheim.

18. Thushan Ekneligoda, *Estimating the Elastic Moduli of Porous Materials from Image Analysis of the Pore Space*, Royal Institute of Technology, December 2007. Opponent: Igor Tsukrov, University of New Hampshire.

19. Fuguo Tong, *Numerical Modelling of Coupled Thermo-Hydro-Mechanical Processes in Geological Porous Media,* (co-supervisor: Lanru Jing), Royal Institute of Technology, March 2010. Opponent: Chin-Fu Tsang, Lawrence Berkeley National Laboratory.

20. Emmanuel David, *The Effect of Stress, Pore Fluid and Pore Structure on Elastic Wave Velocities in Sandstones*, Imperial College, March 2012. Examiner: Michael King.

21. Colin Leung, *Modelling the Flow and Transport Properties of Two-Dimensional Fracture Networks, including the Effect of Stress*, Imperial College, November 2012. Examiner: Dave Sanderson, University of Southampton.

Journal Articles

1. "Elastic moduli of a solid with spherical pores: new self-consistent method", R. W. Zimmerman, *Int. J. Rock Mech.*, vol. 21, pp. 339-343, 1984.

2. "Compressibility of an isolated spheroidal cavity in an isotropic elastic medium", R. W. Zimmerman, *J. Appl. Mech.*, vol. 52, pp. 606-608, 1985.

3. "The effect of microcracks on the elastic moduli of brittle materials", R. W. Zimmerman, *J. Mater. Sci. Letts.*, vol. 4, pp. 1457-1460, 1985.

4. "Comment on 'The constitutive theory for fluid-filled porous materials', by N. Katsube", R. W. Zimmerman, *J. Appl. Mech.*, vol. 52, p. 983, 1985.

5. "The effect of the extent of freezing on seismic velocities in unconsolidated permafrost", R. W. Zimmerman and M. S. King, *Geophysics*, vol. 51, pp. 1285-1290, 1986.

6. "Elastic moduli of mortar as a porous-granular material", R. W. Zimmerman, M. S. King, and P. J. M. Monteiro, *Cement Concr. Res.*, vol. 16, pp. 239-245, 1986.

7. "Compressibility of two-dimensional cavities of various shapes", R. W. Zimmerman, *J. Appl. Mech.*, vol. 53, pp. 500-504, 1986.

8. "Compressibility of porous rocks", R. W. Zimmerman, W. H. Somerton, and M. S. King, *J. Geophys. Res.*, vol. 91, pp. 12765-12778, 1986.

9. "Stress singularity around two nearby holes", R. W. Zimmerman, *Mech. Res. Comm.*, vol. 15, pp. 87-90, 1988.

10. "Seismic and electrical properties of unconsolidated permafrost", M. S. King, R. W. Zimmerman, and R. F. Corwin, *Geophys. Prospect.*, vol. 36, pp. 349-364, 1988.

11. "Stress concentration around a pair of circular holes in a hydrostatically stressed elastic sheet", R. W. Zimmerman, *J. Appl. Mech.*, vol. 55, pp. 477-478, 1988.

12. "Second-order approximation for the compression of an elastic plate containing a pair of circular holes", R. W. Zimmerman, *Zeit. Ang. Math. Mech.*, vol. 68, pp. 575-578, 1988.

13. "Thermal conductivity of fluid-saturated rocks", R. W. Zimmerman, *J. Petrol. Sci. Eng.*, vol. 3, pp. 219-227, 1989.

14. "An approximate solution for one-dimensional absorption in unsaturated porous media", R. W. Zimmerman and G. S. Bodvarsson, *Water Resour. Res.*, vol. 25, pp. 1422-1428, 1989.

15. "Integral method solution for diffusion into a spherical block", R. W. Zimmerman and G. S. Bodvarsson, *J. Hydrol.*, vol. 111, pp. 213-224, 1989.

16. "Absorption of water into porous blocks of various shapes and sizes", R. W. Zimmerman, G. S. Bodvarsson, and E. M. Kwicklis, *Water Resour. Res.*, vol. 26, pp. 2797-2806, 1990.

17. "A simple approximate solution for absorption into a Brooks-Corey medium", R. W. Zimmerman and G. S. Bodvarsson, *Transp. Porous Media*, vol. 6, pp. 195-205, 1991.

18. "Permeability of a fracture with cylindrical asperities", S. Kumar, R. W. Zimmerman, and G. S. Bodvarsson, *Fluid Dyn. Res.*, vol. 7, pp. 131-137, 1991.

19. "Comment on 'Application of linear elastic fracture mechanics to the quantitative evaluation of fluid inclusion decrepitation', by A. Lacazette", S. J. Martel and R. W. Zimmerman, *Geology*, vol. 19, pp. 663-664, 1991.

20. "Reply to "Comment on 'An approximate solution for one-dimensional absorption in unsaturated porous media', by R. W. Zimmerman and G. S. Bodvarsson", by Parlange et al.", R. W. Zimmerman and G. S. Bodvarsson, *Water Resour. Res.*, vol. 27, pp. 2161-2162, 1991.

21. "Lubrication theory analysis of the permeability of rough-walled fractures", R. W. Zimmerman, S. Kumar, and G. S. Bodvarsson, *Int. J. Rock Mech.*, vol. 28, pp. 325-331, 1991.

22. "Elastic moduli of a solid containing spherical inclusions", R. W. Zimmerman, *Mech. of Maters.*, vol. 12, pp. 17-24, 1991.

23. "The effect of contact area on the permeability of fractures", R. W. Zimmerman, D. W. Chen, and N. G. W. Cook, *J. Hydrol.*, vol. 139, pp. 79-96, 1992.

24. "Hashin-Shtrikman bounds on the Poisson ratio of a composite material", R. W. Zimmerman, *Mech. Res. Comm.*, vol. 19, pp. 563-569, 1992.

25. "A numerical dual-porosity model with semi-analytical treatment of fracture/matrix flow", R. W. Zimmerman, G. Chen, T. Hadgu, and G. S. Bodvarsson, *Water Resour. Res.*, vol. 29, pp. 2127-2137, 1993.

26. "Behavior of the Poisson ratio of a two-phase composite material in the high-concentration limit", R. W. Zimmerman, *Appl. Mech. Rev.*, vol. 47, pp. S38-44, 1994.

27. "Accuracy and efficiency of a semi-analytical dual-porosity simulator for flow in unsaturated fractured rock masses", R. W. Zimmerman, G. Chen, and T. Hadgu, *Rad. Waste Manag. Environ. Restor.*, vol. 19, pp. 193-208, 1994.

28. "Grain and void compression in fractured and porous rocks", R.W. Zimmerman, L. R. Myer, and N. G. W. Cook, *Int. J. Rock Mech.*, vol. 31, pp. 179-184, 1994.

29. "Coupled reservoir-wellbore simulation of geothermal reservoir behavior", T. Hadgu, R. W. Zimmerman, and G. S. Bodvarsson, *Geothermics*, vol. 24, pp. 145-166, 1995.

30. "Effective block size for imbibition and absorption in dual-porosity media", R. W. Zimmerman and G. S. Bodvarsson, *Geophys. Res. Letts.*, vol. 22, pp. 1461-1464, 1995.

31. "Thermal stresses and effective thermal expansion coefficient of a functionally-gradient sphere", M. P. Lutz and R. W. Zimmerman, *J. Thermal Stresses*, vol. 19, pp. 39-54, 1996.

32. "Effective transmissivity of a two-dimensional fracture network", R. W. Zimmerman and G. S. Bodvarsson, *Int. J. Rock Mech.*, vol. 33, pp. 433-438, 1996.

33. "Hydraulic conductivity of rock fractures", R. W. Zimmerman and G. S. Bodvarsson, *Transp. Porous Media*, vol. 23, pp. 1-30, 1996.

34. "Effective conductivity of a two-dimensional medium containing elliptical inclusions", R. W. Zimmerman, *Proc. Roy. Soc. London, Series A*, vol. 452, pp. 1713-1727, 1996.

35. "A new lumped-parameter model for flow in unsaturated dual-porosity media", R. W. Zimmerman, T. Hadgu, and G. S. Bodvarsson, *Adv. Water Resour.*, vol. 19, pp. 317-327, 1996.

36. "Effect of the interphase zone on the bulk modulus of a particulate composite", M. P. Lutz and R. W. Zimmerman, *J. Appl. Mech.*, vol. 63, pp. 855-861, 1996.

37. "Inhomogeneous interfacial transition zone model for the bulk modulus of mortar", M. P. Lutz, P. J. M. Monteiro, and R. W. Zimmerman, *Cement Concr. Res.*, vol. 27, pp. 1113-1122, 1997.

38. "The fractal dimension of pores in sedimentary rocks and its influence on permeability", E. M. Schlueter, R. W. Zimmerman, P. A. Witherspoon, and N. G. W. Cook, *Eng. Geol.*, vol. 48, pp. 199-215, 1997.

39. "Formula for the conductivity of a two-component material based on the reciprocity theorem", J. A. del Rio, R. W. Zimmerman, and R. A. Dawe, *Solid State Commun.*, vol. 106, pp. 183-186, 1998.

40. "Effect of shear displacement on the aperture and permeability of a rock fracture", I. W. Yeo, R. W. Zimmerman, and M. H. deFreitas, *Int. J. Rock Mech.*, vol. 35, pp. 1051-70, 1998.

41. "Thermal stresses and thermal expansion in a uniformly-heated functionally-graded cylinder", R. W. Zimmerman and M. P. Lutz, *J. Thermal Stresses*, vol. 22, pp. 177-188, 1999.

42. "Coupling in poroelasticity and thermoelasticity", R. W. Zimmerman, *Int. J. Rock Mech.*, vol. 37, pp. 79-87, 2000.

43. "Effect of stress on the hydraulic conductivity of rock pores", S. Sisavath, X. D. Jing, and R. W. Zimmerman, *Phys. Chem. Earth*, vol. 25, pp. 163-168, 2000.

44. "A model for steady laminar flow through a deformable gel-coated channel", C. Yang, C. A. Grattoni, A. H. Muggeridge, and R. W. Zimmerman, *J. Colloid Interface Sci.*, vol. 226, pp. 105-111, 2000.

45. "Flow of oil and water through elastic polymer gels", H. H. Al-Sharji, C. A. Grattoni, R. A. Dawe, and R. W. Zimmerman, *Oil & Gas Sci. Tech. - Rev. IFP*, vol. 56, pp. 145-152, 2001.

46. "Laminar flow through irregularly-shaped pores in sedimentary rocks", S. Sisavath, X. D. Jing, and R. W. Zimmerman, *Transp. Porous Media*, vol. 45, pp. 41-62, 2001.

47. "Accuracy of the renormalization method for computing effective conductivities of heterogeneous media", I. W. Yeo and R. W. Zimmerman, *Transp. Porous Media*, vol. 45, pp. 129-138, 2001.

48. "Rheology and permeability of crosslinked polyacrylamide gel", C. A. Grattoni, H. H. Al-Sharji, C. Yang, A. H. Muggeridge, and R. W. Zimmerman, *J. Colloid Interface Sci.*, vol. 240, pp. 601-607, 2001.

49. "Creeping flow through a pipe of varying radius", S. Sisavath, X. D. Jing, and R. W. Zimmerman, *Phys. Fluids*, vol. 13, pp. 2762-2772, 2001.

50. "Creeping flow through an axisymmetric sudden contraction or expansion", S. Sisavath, X. D. Jing, C. C. Pain, and R. W. Zimmerman, *ASME J. Fluids Eng.*, vol. 124, pp. 273-278, 2002.

51. "Wettability alteration by aging of a gel placed within a porous medium", C. A. Grattoni, X. D. Jing, and R. W. Zimmerman, *J. Petrol. Sci. Eng.*, vol. 33, pp. 135-145, 2002.

52. "Flow of water through channels filled with deformable polymer gels", C. Yang, C. A. Grattoni, A. H. Muggeridge, and R. W. Zimmerman, *J. Colloid Interface Sci.*, vol. 250, pp. 466-470, 2002.

53. "Segregated pathways mechanism for oil and water flow through silicate gels formed from an oil-based gelant", C. A. Grattoni, H. H. Al-Sharji, R. A. Dawe, and R. W. Zimmerman, *J. Petrol. Sci. Eng.*, vol. 35, pp. 183-190, 2002.

54. "Predicting the permeability of sandstone from image analysis of pore structure", P. A. Lock, X. D. Jing, R. W. Zimmerman, and E. M. Schlueter, *J. Appl. Phys.*, vol. 92, pp. 6311-6319, 2002.

55. "A simple model for deviations from the cubic law for a fracture undergoing dilation or closure", S. Sisavath, A. Al-Yaarubi, C. C. Pain, and R. W. Zimmerman, *Pure Appl. Geophys. (PAGEOPH)*, vol. 160, pp 1009-1022, 2003.

56. "Laplace transform inversion for late-time behavior of groundwater flow problems", S. A. Mathias and R. W. Zimmerman, *Water Resour. Res.*, vol. 39, paper 1283, 2003.

57. "Comparison of methods for upscaling permeability from the pore scale to the core scale", P. A. Lock, X. D. Jing, and R. W. Zimmerman, *J. Hydraul. Res.*, vol. 42, pp. 3-8, 2004.

58. "Effective stress law for the permeability of clay-rich sandstones", W. Al-Wardy and R. W. Zimmerman, *J. Geophys. Res.*, vol. 109, No. B4, B04203, 2004.

59. "Nonlinear regimes of fluid flow in rock fractures", R. W. Zimmerman, A. H. Al-Yaarubi, C. C. Pain, and C. A. Grattoni, *Int. J. Rock Mech.*, vol. 41, paper 1A27, p. 163, 2004.

60. "Polymers as relative permeability modifiers: adsorption and the dynamic force of thick polyacrylamide layers", C. A. Grattoni, P. F. Luckham, X. D. Jing, L. Norman, and R. W. Zimmerman, *J. Petrol. Sci. Eng.*, vol. 45, pp. 233-245, 2004.

61. "Effect of an inhomogeneous interphase zone on the bulk modulus and conductivity of a particulate composite", M. P. Lutz and R. W. Zimmerman, *Int. J. Solids Struct.*, vol. 42, pp. 429-437, 2005.

62. "Relation between the Mogi and the Coulomb failure criteria", A. M. Al-Ajmi and R. W. Zimmerman, *Int. J. Rock Mech.*, vol. 42, pp. 431-39, 2005.

63. "Analytical analysis for oil recovery during counter-current imbibition", Z. Tavassoli, R. W. Zimmerman, and M. J. Blunt, *Transp. Porous Media*, vol. 58, pp. 173-189, 2005.

64. "Analysis of counter-current imbibition with gravity in weakly water-wet systems", Z. Tavassoli, R. W. Zimmerman, and M. J. Blunt, *J. Petrol. Sci. Eng*, vol. 48, pp. 94-104, 2005.

65. "Compressibility of two-dimensional pores having n-fold axes of symmetry", T. C. Ekneligoda and R. W. Zimmerman, *Proc. Roy. Soc. London, Ser. A*, vol. 462, pp. 1933-1947, 2006.

66. "Pore-scale modelling of NMR relaxation for the characterization of wettability", S. H. Al-Mahrooqi, C. A. Grattoni, A. H. Muggeridge, R. W. Zimmerman and X. D. Jing, *J. Petrol. Sci. Eng.*, vol. 52, pp. 172-186, 2006.

67. "Stability analysis of vertical boreholes using the Mogi-Coulomb failure criterion", A. M. Al-Ajmi and R. W. Zimmerman, *Int. J. Rock Mech.*, vol. 43, pp. 1200-1211, 2006.

68. "Thermal and electrical conductivity of composites with graded interfaces", R. W. Zimmerman and M. P. Lutz, *Adv. Sci. Tech.*, vol. 45, pp. 1097-1102, 2006.

69. "Influence of volume/mass on grain-size curves and conversion of image-analysis size to sieve size", J. M. R. Fernlund, R. W. Zimmerman, and D. Kragic, *Eng. Geol.*, vol. 90, pp. 124-137, 2007.

70. "Boundary perturbation solution for nearly-circular holes and rigid inclusions in an infinite elastic medium", T. C. Ekneligoda and R. W. Zimmerman, *J. Appl. Mech.*, vol. 75, paper 011015-1, 2008.

71. "Shear compliance of two-dimensional pores possessing N-fold axis of rotational symmetry", T. C. Ekneligoda and R. W. Zimmerman, *Proc. Roy. Soc. London, Ser. A*, vol. 464, pp. 759-775, 2008.

72. "Sensitivity of the impact of geological uncertainty on production from faulted and unfaulted shallow-marine oil reservoirs: objectives and methods", T. Manzocchi, J. N. Carter,....R. W. Zimmerman, *Petrol. Geosci.*, vol. 14, pp. 3-15, 2008.

73. "Assessing the effect of geological uncertainty on recovery estimates in shallow-marine reservoirs: the application of reservoir engineering to the SAIGUP project", J. D. Matthews, J. N. Carter, K. D. Stephen, R. W. Zimmerman, A. Skorstad, T. Manzocchi, J.A. Howell, *Petrol. Geosci.*, vol. 14, pp. 35-44, 2008.

74. "Using drill-stem and production tests to model reservoir relative permeabilities", J. D. Matthews, J. N. Carter, and R. W. Zimmerman, *SPE Reserv. Eval. & Eng.*, vol 11, pp. 1082-1088, 2008.

75. "Approximate solutions for pressure buildup during CO2 injection in brine aquifers", S. A. Mathias, P. E. Hardisty, M. R. Trudell, and R. W. Zimmerman, *Transp. Porous Media*, vol. 79, pp. 269-284, 2009.

76. "Screening and selection of sites for CO2 sequestration based on pressure buildup", S. A. Mathias, P. E. Hardisty, M. R. Trudell, and R. W. Zimmerman, *Int. J. Greenhouse Gas Control*, vol. 3, pp. 577-585, 2009.

77. "A new well path optimization model for increased mechanical borehole stability", A. M. Al-Ajmi and R. W. Zimmerman, *J. Petrol. Sci. Eng.*, vol. 69, pp. 53-62, 2009.

78. "Influence of the interfacial transition zone and microcracking on the diffusivity, permeability and sorptivity of cement-based materials after drying", H. Wong, M. Zobel, N. Buenfeld, and R. W. Zimmerman, *Mag. Concr. Res.*, vol. 61, pp. 571-589, 2009.

79. "An effective thermal conductivity model of geological porous media for coupled thermo-hydro-mechanical systems with multiphase flow", F. G. Tong, L. Jing, and R. W. Zimmerman, *Int. J. Rock Mech.*, vol. 46, pp. 1358-1369, 2009.

80. "A fully coupled thermo-hydro-mechanical model for simulating multiphase flow, deformation and heat transfer in buffer material and rock masses", F. G. Tong, L. Jing, and R. W. Zimmerman, *Int. J. Rock Mech.*, vol. 47, pp. 205-217, 2010.

81. "Compressibility and shear compliance of spheroidal pores: Exact derivation via the Eshelby tensor, and asymptotic expressions in limiting cases", E. C. David and R. W. Zimmerman, *Int. J. Solids Struct.*, vol. 48, pp. 680-686, 2011.

82. "Numerical simulation of multiple 3D fracture propagation using arbitrary meshes", A. Paluszny and R. W. Zimmerman, *Comp. Meth. Appl. Mech. Eng.*, vol. 200, pp. 953-966, 2011.

83. "Elastic moduli of solids containing spheroidal pores", E. C. David and R. W. Zimmerman, *Int. J. Eng. Sci.*, vol. 49, pp. 544-560, 2011.

84. "Effect of entrained air voids on the microstructure and mass transport properties of concrete", H. S. Wong, A. M. Pappas, R.W. Zimmerman, and N. R. Buenfeld, *Cem. Concr. Res.*, vol. 41, pp. 1067-1077, 2011.

85. "Pressure buildup during CO2 injection into a closed brine aquifer", S. A. Mathias, G. J. González, K. E. Thatcher, and R. W. Zimmerman, *Transp. Porous Media*, vol. 89, pp. 383-397, 2011.

86. "Estimating the permeability of cement pastes and mortars using image analysis and effective medium theory", H. S. Wong, R. W. Zimmerman, and N. R. Buenfeld, *Cem. Concr. Res.*, vol. 42, pp. 476-483, 2012.

87. "Sliding crack model for nonlinearity and hysteresis in the uniaxial stress-strain curve of rock", E. C. David, N. Brantut, A. Schubnel, and R. W. Zimmerman, *Int. J. Rock Mech.*, vol. 52, pp. 9-17, 2012.

88. "Estimating the hydraulic conductivity of two-dimensional fracture networks using network geometric properties", C. T. O. Leung and R. W. Zimmerman, *Transp. Porous Media*, vol. 97, pp. 777–797, 2012.

89. "Pore structure model for elastic wave velocities in fluid saturated sandstones", E. C. David and R. W. Zimmerman, *J. Geophys. Res.*, vol. 117, paper B07210, 2012.

90. "Comparison of discrete fracture network and equivalent continuum simulations of fluid flow through two-dimensional fracture networks for the DECOVALEX-2011 project", C. T. O. Leung, A. R. Hoch, and R. W. Zimmerman, *Mineralogical Mag.*, vol. 76, pp. 3179-3190, 2012.

Proceedings, Book Chapters, Reviews, etc.

1. "Compressibilities and effective stress coefficients for linear elastic porous solids", R. W. Zimmerman, in *Proc. 23rd U.S. Rock Mech. Symp.,* Soc. Mining Eng., New York, pp. 712-729, 1982.

2. "The effects of pore pressure and confining pressure on the pore and bulk compressibilities of consolidated sandstones", R. W. Zimmerman, J. L. Haraden, and W. H. Somerton, in *Measurement of Rock Properties at Elevated Temperatures and Pressures*, ASTM Spec. Tech. Pub. 869, Amer. Soc. Test. Maters., Philadelphia, pp. 23-35, 1985.

3. "Propagation of acoustic waves through cracked rock", R. W. Zimmerman and M. S. King, in *Proc. 26th U.S. Rock Mech. Symp.*, A.A. Balkema, Rotterdam, pp. 739-745, 1985.

4. "Compressibility of two-dimensional holes of various shapes", R. W. Zimmerman, in *Proc. 19th Midwest. Mech. Conf.*, Ohio State University, Columbus, Ohio, pp. 259-260, 1985.

5. "Seismic velocities in unconsolidated permafrost", R. W. Zimmerman, M. S. King, and R. F. Corwin, in *Proc. 27th U.S. Rock Mech. Symp.,* Soc. Mining Eng., New York, pp. 640-645, 1986.

6. "Compression of an elastic plate containing a pair of circular holes", R. W. Zimmerman, in *Proc. 20th Midwest. Mech. Conf.*, Purdue University, West Lafayette, Ind., pp. 1091-1096, 1987.

7. "The effect of contact area on the permeability of fractures", D. W. Chen, R. W. Zimmerman, and N. G. W. Cook, in *Proc. 30th U.S. Rock Mech. Symp.*, A. A. Balkema, Rotterdam, pp. 81-88, 1989.

8. "Semi-analytical solutions for flow problems in unsaturated porous media", R. W. Zimmerman and G. S. Bodvarsson, in *Multiphase Transport in Porous Media,* ASME Pub. FED-82, Amer. Soc. Mech. Eng., New York, pp. 23-28, 1989.

9. "Combined analytical/numerical approaches to solving fluid flow problems in the unsaturated zone at Yucca Mountain", R. W. Zimmerman and G. S. Bodvarsson, in *Proc. 1st Int. High-Level Rad. Waste Manag. Conf.*, Amer. Nucl. Soc., pp. 818-24, 1990.

10. "Study of fractal aperture distribution and flow in fractures", S. Kumar, R. W. Zimmerman, and G. S. Bodvarsson, in *Proc. 1st Int. High-Level Rad. Waste Manag. Conf.*, Amer. Nucl. Soc., pp. 559-564, 1990.

11. "Hydromechanical coupling between stress, stiffness, and hydraulic conductivity of rock joints and fractures", R. W. Zimmerman, D. W. Chen, J. C. S. Long, and N. G. W. Cook, in *Proc. Int. Conf. Rock Joints*, A. A. Balkema, Rotterdam, pp. 571-577, 1990.

12. "Predicting the permeability and electrical conductivity of sedimentary rocks from microstructure", R. W. Zimmerman, E. M. Schlueter, N. G. W. Cook, and P. A. Witherspoon, in *Proc. 32nd U.S. Rock Mech. Symp.*, A. A. Balkema, Rotterdam, pp. 355-364, 1991.

13. "Lubrication theory analysis of the permeability of rough-walled fractures", R. W. Zimmerman, S. Kumar, and G. S. Bodvarsson, in *Proc. 2nd Int. High-Level Rad. Waste Manag. Conf.*, Amer. Nucl. Soc., pp. 535-541, 1991.

14. "A fluid-mechanical model for blood flow in lung alveoli", R. W. Zimmerman and S. Kumar, in *Advances in Biological Heat and Mass Transfer*, ASME Pub. BED-18, Amer. Soc. Mech. Eng., New York, pp. 51-56, 1991.

15. "Approximate expressions for stress concentrations in elastic solids containing multiple hole or inclusions", R. W. Zimmerman, in *Recent Developments in Elasticity*, Pub. AMD-24, Amer. Soc. Mech. Eng., New York, pp. 71-76, 1991.

16. "A dual-porosity reservoir model with an improved coupling term", R. W. Zimmerman, G. Chen, and G. S. Bodvarsson, in *Proc. 17th Workshop Geothermal Reservoir Eng.*, Stanford University, Stanford, Calif., pp. 159-164, 1992.

17. "Integral method solutions to flow problems in unsaturated porous media", R. W. Zimmerman and G. S. Bodvarsson, in *Modeling and Analysis of Diffusive and Advective Processes in Geoscience*, Society for Industrial and Applied Mathematics, Philadelphia, pp. 222-233, 1992.

18. "Semi-analytical treatment of fracture/matrix flow in a dual-porosity simulator for unsaturated fractured rock masses", R. W. Zimmerman and G. S. Bodvarsson, in *Proc. 3rd Int. High-Level Rad. Waste Manag. Conf.*, Amer. Nucl. Soc., pp. 272-278, 1992.

19. "A dual-porosity reservoir model with a nonlinear coupling term", R. W. Zimmerman, G. Chen, T. Hadgu, and G. S. Bodvarsson, in *Proc. Int. Conf. Flow through Porous Media,* Russian Academy of Sciences, Moscow, pp. 100-103, 1992.

20. "Network model of sandstone permeability", E. M. Schlueter and R. W. Zimmerman, in *Composite Material Technology,* ASME Pub. PD-53, Amer. Soc. Mech. Eng., New York, pp. 207-212, 1993.

21. "An inverse procedure for estimating the unsaturated hydraulic conductivity of volcanic tuff", R. W. Zimmerman, G. S. Bodvarsson, A. L. Flint, and L. E. Flint, in *Proc. 4th Int. High-Level Rad. Waste Manag. Conf.,* Amer. Nucl. Soc., pp. 1052-1057, 1993.

22. "Development of a dual-porosity model for vapor-dominated fractured geothermal reservoirs", R. W. Zimmerman, T. Hadgu, and G. S. Bodvarsson, in *Proc. 18th Workshop Geothermal Reservoir Eng.,* Stanford University, Stanford, Calif., pp. 279-284, 1993.

23. "Coupling of a reservoir simulator and a wellbore simulator for geothermal applications", T. Hadgu, R. W. Zimmerman, and G. S. Bodvarsson, in *Transactions, Vol. 17,* Geothermal Resources Council, Davis, Calif., pp. 499-505, 1993.

24. "Theoretical studies of flowrates from slimholes and production-size geothermal wells", T. Hadgu, R. W. Zimmerman, and G. S. Bodvarsson, in *Proc. 19th Workshop Geothermal Reservoir Eng.,* Stanford University, Stanford, Calif., pp. 253-260, 1994.

25. "Approximating the imbibition and absorption behavior of a distribution of matrix blocks by an equivalent spherical block", R. W. Zimmerman and G. S. Bodvarsson, in *Proc. 5th Int. High-Level Rad. Waste Manag. Conf.,* Amer. Nucl. Soc., pp. 2030-2037, 1994.

26. "Predicting the capillary pressure function of Berea sandstone from microgeometry", E. M. Schlueter, R. W. Zimmerman, L. R. Myer, N. G. W. Cook, and P. A. Witherspoon, in *Comp. Meth. & Adv. Geomech.,* A. A. Balkema, Rotterdam, pp. 1257-1262, 1994.

27. "A simple procedure for estimating the effective hydraulic conductivity of a two-dimensional saturated or partly-saturated fracture network", R. W. Zimmerman and G. S. Bodvarsson, in *Proc. 6th Int. High-Level Rad. Waste Manag. Conf.,* Amer. Nucl. Soc., pp. 26-28, 1995.

28. "Design and analysis of an experimental apparatus to measure directional permeabilities of a rock fracture under normal and shear loading", I. W. Yeo, R. W. Zimmerman, and M. H. deFreitas, in *Proc. Eur. Rock Mech. Symp.,* "EUROCK '96", A.A. Balkema, Rotterdam, pp. 1223-1227, 1996.

29. "Inhomogeneous interfacial transition zone model for the elastic moduli of concrete", M. P. Lutz, P. J. M. Monteiro, and R. W. Zimmerman, in *Proc. 4th Maters. Eng. Conf.*, Amer. Soc. Civil. Eng., New York, pp. 1246-1255, 1996.

30. "Thermal stresses in a functionally-graded cylinder", M. P. Lutz and R. W. Zimmerman, in *Proc. 2nd Int. Symp. Thermal Stresses,* R. B. Hetnarski and N. Noda, eds., Rochester Institute of Technology, Rochester, N.Y., pp. 323-326, 1997.

31. "The effect of shear displacement on the void geometry and aperture distribution of a rock fracture", I. W. Yeo, R. W. Zimmerman, and M. H. deFreitas, in *Proc. 3rd Int. Symp. Mech. of Jointed & Fractured Rock,* A. A. Balkema, Rotterdam, pp. 223-228, 1998.

32. "Pore-scale study of the flow of oil and water through polymer gels", H. H. Al-Sharji, C. A. Grattoni, R. A. Dawe, and R. W. Zimmerman, in *Proc. 1999 SPE Ann. Tech. Conf.*, Soc. Pet. Eng., Dallas, paper SPE 56738, 1999.

33. Review of *Fractures and Fracture Networks*, by P. M. Adler and J.-F. Thovert, in *Transport in Porous Media*, vol. 38, pp. 355-356, 2000.

34. "Micromechanics of poroelastic rocks", R. W. Zimmerman, in *Heterogeneous Media: Modelling, Methods, and Simulation,* K. Z. Markov and L. Preziosi, eds., Birkhauser, Boston, pp. 409-460, 2000.

35. "Fluid flow in rock fractures: From the Navier-Stokes equations to the cubic law", R. W. Zimmerman and I. W. Yeo, in *Dynamics of Fluids in Fractured Rocks*, B. Faybishenko, S. Benson, and P. Witherspoon, eds., Amer. Geophys. Union, Washington, pp. 213-224, 2000.

36. "Influence of geological features on well test behavior", J. Zambrano, R. W. Zimmerman, and A. C. Gringarten, in *Proc. 2000 SPE Asia Pacific Conf.*, Soc. Pet. Eng., Dallas, paper SPE 59398, 2000.

37. "Implications of static poroelasticity for reservoir compaction", R. W. Zimmerman, in *Proc. 4th North Amer. Rock Mech. Symp.*, A. A. Balkema, Rotterdam, pp. 169-172, 2000.

38. "Pore compressibility under uniaxial strain", R. W. Zimmerman, in *Proc. 6th Int. Symp. Land Subsidence*, National Research Council of Italy, Venice, pp. 57-65, 2000.

39. "Improved reservoir characterization through evolutionary computation", C. E. Romero, J. N. Carter, R. W. Zimmerman, and A. C. Gringarten, in *Proc. 2000 SPE Ann. Tech. Conf. Exh.*, Soc. Pet. Eng., Dallas, paper SPE 62942, 2000.

40. "Flow of water through elastically-deformable polymer gels", C. Yang, C. A. Grattoni, A. Muggeridge, and R. W. Zimmerman, in *ECMOR VII: Proc. 7th Eur. Conf. Math. Oil Recov.*, Eur. Ass. Geosci. Eng., The Hague, paper V-13, 2000.

41. "A modified genetic algorithm for reservoir characterisation", C. E. Romero, J. N. Carter, A. C. Gringarten, and R. W. Zimmerman, in *Proc. 2000 SPE Int. Oil Gas Conf.*, Soc. Pet. Eng., Dallas, paper SPE 64765, 2000.

42. "A simple model for deviations from the 'cubic law' for a fracture under normal stress", S. Sisavath and R. W. Zimmerman, in *Proc. 3rd Euroconf. Rock Phys. Rock Mech.*, Univ. Bonn, paper 35, 2000.

43. "Dimensionless coupling parameters in poroelasticity and thermoelasticity", R. W. Zimmerman, in *Proc. 3rd Euroconf. Rock Phys. Rock Mech.*, Univ. Bonn, paper 51, 2000.

44. "Relative permeability modification", M. Bartosek, C. A. Grattoni, A. Stavland, and R. W. Zimmerman, in *Well Treatment and Water Shutoff by Polymer Gels*, P. L. J. Zitha, ed., Delft University Press, Delft, pp. 1-117, 2000.

45. "Disproportionate permeability reduction when a silicate gel is formed *in-situ* to control water production", C. A. Grattoni, X. D. Jing, and R. W. Zimmerman, in *Proc. SPE Latin Amer. Pet. Eng. Conf.*, Soc. Pet. Eng., Dallas, paper SPE 69534, 2001.

46. "Use of image analysis to study the effect of pore roughness on the hydraulic conductivity of reservoir rocks", S. Sisavath, P. A. Lock, X. D. Jing, and R. W. Zimmerman, in *Proc. 5th Nordic Symp. Petrophys.* (Copenhagen, 19-20 Aug. 1999), Nordisk Energiforskning, pp. 149-156, 2001.

47. "Disproportionate permeability reduction due to polymer adsorption entanglement", H. H. Al-Sharji, C. A. Grattoni, R. A. Dawe, and R. W. Zimmerman, in *Proc. SPE Eur. Form. Damage Conf.*, Soc. Pet. Eng., Dallas, paper SPE 68972, 2001.

48. "Relative permeability modification using an oil-soluble gelant to control water", G. P. Karmakar, C. A. Grattoni, and R. W. Zimmerman, in *Proc. 2002 SPE Int. Oil Gas Conf.*, Soc. Pet. Eng., Dallas, paper SPE 77414, 2002.

49. "Effective stress laws for the permeability of clay-rich sandstones", W. Al-Wardy and R. W. Zimmerman, in *Proc. 2002 Int. Conf. Soc. Core Analysts*, Monterey, Calif., 20-23 Sept., paper A45, 2002.

50. "The influence of clay fraction on the complex impedance of shaly sands", R. Al-Mjeni, F. Günzel, X. D. Jing, C. A. Grattoni, and R. W. Zimmerman, in *Proc. 2002 Int. Conf. Soc. Core Analysts*, Monterey, Calif., 20-23 Sept., paper A44, 2002.

51. "Functional parameters for quantifying the surface anisotropy of rock discontinuities", G. Grasselli, J. Wirth, and R. W. Zimmerman, in *Proc. Int. Symp. Rock Eng. Mountainous Regions*, "EUROCK 2002", Madeira, Portugal, A. A. Balkema, Rotterdam, pp. 715-721, 2002.

52. "Effect of an inhomogeneous interphase zone on the elastic and thermal properties of a particulate composite, R. W. Zimmerman and M. P. Lutz, in *Proc. 9th Int. Conf. Mech. Behav. Maters.*, Geneva, 25-29 May 2003.

53. "Effective stress law for the permeability of clay-rich sandstones", W. Al-Wardy and R. W. Zimmerman, in *Proc. 16th Eng. Mech. Conf. Amer. Soc. Civil Eng.*, Seattle, 16-18 July 2003. (CD)

54. "Flow behaviour and gel characteristics of an oil-based system", G. P. Karmakar, C. A. Grattoni, and R. W. Zimmerman, in *10th Indian Oil Gas Review Symp.*, Mumbai, 8-9 Sept. 2003, paper 68, 2003.

55. "Surface parameters for quantifying the hydro-mechanical anisotropy of rock discontinuities", G. Grasselli, J. Wirth, and R. W. Zimmerman, in *Proc. 10th Cong. Int. Soc. Rock Mech.*, Sandton, South Africa, 8-12 September 2003, SAIMM, pp. 415-423, 2003.

56. "Hydromechanical behaviour of fractured rocks", R. W. Zimmerman and I. G. Main, in *Mechanics of Fluid-Saturated Rocks*, Y. Guéguen and M. Boutéca, eds., Elsevier Academic Press, San Diego and London, pp. 363-421, 2004.

57. "Navier-Stokes simulations of fluid flow through a rock fracture", A. H. Al-Yaarubi, C. C. Pain, C. A. Grattoni and R. W. Zimmerman, *Proc. 2nd Int. Symp. Dynamics of Fluids in Fractured Rocks,* Lawrence Berkeley National Laboratory Report LBNL-54275, pp. 201-205, 2004.

58. "Skempton coefficient of clean and clay-rich sandstones", W. Al-Wardy and R. W. Zimmerman, in *Proc. 17th Eng. Mech. Conf. Amer. Soc. Civil Eng.*, Newark, Del., 13-16 June 2004. (CD)

59. "Estimating the permeability of carbonate rocks using image analysis and effective medium theory", M. Jurgawczynski, R. W. Zimmerman, and X. D. Jing, in *Proc. 2004 Int. Conf. Soc. Core Analysts*, Abu-Dhabi, 5-9 Oct., paper A30, 2004.

60. "Fluid flow in rock fractures", R. W. Zimmerman, in *Proc. 11th Int. Conf. Int. Assoc. Comp. Methods Appl. Geomech.*, Turin, 19-25 June, vol. 4, pp. 89-107, 2005.

61. "Wellbore stability analysis using the Mogi-Coulomb failure criterion", A. Al-Ajmi and R. W. Zimmerman, in *Proc. 40th U. S. Symp. Rock Mech.*, Anchorage, Alaska, paper ARMA 05-784, 25-29 June 2005.

62. "The effect of fracture relative permeabilities and capillary pressures on the numerical simulation of naturally fractured reservoirs", J. J. de la Porte, C. A. Kossack, and R. W. Zimmerman, in *Proc. 2005 SPE Ann. Tech. Conf.*, Dallas, 9-12 October 2005, paper SPE 95241.

63. "Navier-Stokes simulations of fluid flow through a rock fracture", A. H. Al-Yaarubi, C. C. Pain, C. A. Grattoni, and R. W. Zimmerman, in *Dynamics of Fluids and Transport in Fractured Rocks*, B. Faybishenko, P. A. Witherspoon, and J. Gale, eds., Amer. Geophys. Union, Washington, DC, pp. 55-64, 2005.

64. "A new 3D stability model for the design of non-vertical wellbores", A. Al-Ajmi and R. W. Zimmerman, in *41st U. S. Symp. Rock Mech.*, Golden, Colo., paper ARMA 06-961, 18-21 June 2006.

65. "Stability analysis of deviated boreholes using the Mogi-Coulomb failure criterion, with applications to some oil and gas reservoirs", A. Al-Ajmi and R. W. Zimmerman, *Proc. 2006 SPE Asia-Pacific Drilling Tech. Conf.*, Bangkok, 13-15 Nov. 2006, paper SPE 104035, 2006.

66. "Thermal and electrical conductivity of composites with graded interfaces", R. W. Zimmerman and M. P. Lutz, in *Proc. 11th Int. Ceramics Conf.* (Acireale, Sicily, June 2006) *Adv. Sci. Tech.*, vol. 45, pp. 1097-1102, 2006.

67. "Sliding crack model for uniaxial compression of rock", E. David and R. W. Zimmerman, in *Proc. 1st Can.-U.S. Rock Mech. Symp.*, Vancouver, 27-31 May 2007, Taylor & Francis, pp. 575-580, 2007.

68. "Effect of small-scale roughness on the pore compressibility of porous media", T. C. Ekneligoda and R. W. Zimmerman, in *Proc. 18th Eng. ASCE Mech. Conf.*, Blacksburg, Virginia, June 3-6, 2007. (CD)

69. "The Mogi-Coulomb true-triaxial failure criterion, and some implications for rock engineering", A. M. Al-Ajmi and R. W. Zimmerman, in *Proc. 11th Cong. Int. Soc. Rock Mech.*, Lisbon, 7-13 July 2007, Taylor & Francis, pp. 475-479, 2007.

70. "Estimating the elastic moduli of sandstones using two-dimensional pore space images", T. C. Ekneligoda and R. W. Zimmerman, in *Proc. 1st Sri-Lankan Geotech. Soc. Int. Conf. Soil & Rock Eng.*, Colombo, Sri Lanka, August 7-11, 2007. (CD)

71. "Using drill-stem and production tests to model reservoir relative permeabilities", J. D. Matthews, J. N. Carter, and R. W. Zimmerman, in *Proc. Offshore Eur. 2007*, Aberdeen, Scotland, 4-7 Sept. 2007, paper SPE 107209, 2007.

72. "Permeability predictions based on two-dimensional pore space images", M. Jurgawczynski, P. A. Lock, X. D. Jing, and R. W. Zimmerman, in *Proc. 2007 SPE Ann. Tech. Conf.*, Anaheim, 11-14 Nov. 2007, paper SPE 110472, 2007.

73. "A simple model for coupling between the normal stiffness and the hydraulic transmissivity of a fracture", R. W. Zimmerman, in *Proc. 2nd Can.-U.S. Rock Mech. Symp.*, 29 June-02 July 2008, San Francisco, paper ARMA 08-314, 2008. (CD)

74. "Sliding crack model for the uniaxial stress-strain curve of rock", E. David, N. Brantut, A. Schubnel, and R. W. Zimmerman, in *Proc. 1st Southern Hemisphere Rock Mech. Symp.*, Perth, 15-19 Sept. 2008, Australian Centre for Geomechanics, pp. 553-561, 2008.

75. "Modelling multiphase flow, deformation and heat transfer in buffer material", F. G. Tong. L. Jing and R. W. Zimmerman, in *Proc. SinoRock 2009*, 19-22 May 2009, Hong Kong, paper 299. (CD)

76. "On the analogy between poroelasticity and thermoelasticity", R. W. Zimmerman, in *Poromechanics IV: Proc. 4th Maurice A. Biot Conference on Poromechanics*, Columbia University, New York, 8-10 June 2009; DesTech Publications, Lancaster, Penn., pp. 531-536.

77. "A fully-coupled finite element code for modeling thermo-hydro-mechanical processes in porous geological media", F. Tong, L. Jing, and R. W. Zimmerman, in *Proc. 43rd U.S. Rock Mech. Symp.*, 28 June-01 July 2009, Asheville, N.C., paper ARMA 09-120, 2009.

78. "The use of TOUGH-FLAC for Coupled Hydro-Mechanical Modeling of Fractured Rock Masses", C. T. O. Leung, J. Rutqvist, R. W. Zimmerman, and A. R. Hoch, in *Proc. 2009 TOUGH Symp.*, Berkeley, Calif., 14-16 September 2009. (CD)

79. "Estimating the hydraulic conductivity of two-dimensional fracture networks using effective medium theory and power-law averaging", C. T. O. Leung, and R. W. Zimmerman, *in Proc. Eur. Rock Mech. Symp. (Eurock 2010)*, Lausanne, Switzerland, 15-18 June 2010, CRC Press, pp. 243-246, 2010.

80. "Some rock mechanics issues in petroleum engineering", R. W. Zimmerman, in *Proc. Eur. Rock Mech. Symp. (Eurock 2010)*, Lausanne, Switzerland, 15-18 June 2010, CRC Press, pp. 39-44, 2010.

81. "Estimating the permeability of cement-based materials using image analysis and effective medium theory", H. S. Wong, R.W. Zimmerman, and N. R. Buenfeld, in *Proc. RILEM Symp. Concr. Modelling (CONMOD 2010)*, Lausanne, Switzerland, 22-25 June 2010, RILEM, pp. 37-40, 2010.

82. "Shape factors for irregularly shaped matrix blocks", K. Wuthicharn and R. W. Zimmerman, in *Proc. SPE Reservoir Characterization & Simulation Conf.*, Abu Dhabi, 9-11 October 2011, paper SPE 148060, 2011.

83. "A multi-modal approach to 3D fracture and fragmentation of rock using impulse-based dynamics and the finite element method", A. Paluszny, X. H. Tang, and R. W. Zimmerman, in *Proc. 46rd U.S. Rock Mech. Symp.*, 24-27 June 2012, Chicago, paper ARMA 12-213, 2012.

84. “Some new developments in modelling the failure, fracture and fragmentation of rocks”, R. W. Zimmerman and A. Paluszny, in *Proc. 7th Asian Rock Mech. Symp.*, Seoul, 15-17 October 2012 (CD).

85. “Theoretical and numerical modeling of rock hysteresis based on sliding of microcracks”, M. Nejati, A. Paluszny, and R. W. Zimmerman, in *Proc. 47th U.S. Rock Mech. Symp.*, San Francisco, 23-26 June 2013, paper ARMA 13-394, 2013.

Thesis

The Effect of Pore Structure on the Pore and Bulk Compressibilities of Consolidated Sandstones, Department of Mechanical Engineering, University of California, Berkeley, Calif., 1984.

Books

R. W. Zimmerman, *Compressibility of Sandstones* (Developments in Petroleum Science, Vol. 29), Elsevier, Amsterdam, 1991, 173 pp.

J. C. Jaeger, N. G. W. Cook, and R. W. Zimmerman, *Fundamentals of Rock Mechanics*, 4th ed., Blackwell, Oxford, 2007, 475 pp.

Appendix 3
List of Consideration Materials

1. BP-HZN-2179MDL00470598
2. BP-HZN-2179MDL00470599
3. BP-HZN-2179MDL02393271
4. BP-HZN-2179MDL02393883
5. BP-HZN-2179MDL02394182
6. BP-HZN-2179MDL02394183
7. BP-HZN-2179MDL02394184
8. BP-HZN-2179MDL02394185
9. BP-HZN-2179MDL02394186
10. BP-HZN-2179MDL02394187
11. BP-HZN-2179MDL03290054
12. BP-HZN-2179MDL04843794
13. BP-HZN-2179MDL04899278
14. BP-HZN-2179MDL04923119
15. BP-HZN-2179MDL05755276
16. BP-HZN-2179MDL05789875
17. BP-HZN-2179MDL05789876
18. BP-HZN-2179MDL05864773
19. BP-HZN-2179MDL05864804
20. BP-HZN-2179MDL06105310

21. BP-HZN-2179MDL06127378

22. BP-HZN-2179MDL06392037

23. BP-HZN-2179MDL06566208

24. BP-HZN-2179MDL06566258

25. BP-HZN-2179MDL06605384

26. BP-HZN-2179MDL06726208

27. BP-HZN-2179MDL06990570

28. BP-HZN-2179MDL07033640

29. BP-HZN-2179MDL07033640

30. BP-HZN-2179MDL07066668

31. BP-HZN-2179MDL07087480

32. BP-HZN-MBI00023865

33. C. Wolfe, SPE, Baker Atlas; C. Russell and N. Luise, SPE, Eni Petroleum; and R. Chhajlani, SPE, Chevron (2005) Log-Based Pore Volume Compressibility Prediction-A Deepwater GoM Case Study; SPE 95545

34. Chuanliang Li, Xiaofan Chen and Zhimin Du, State Key Laboratory of Oil/Gas Resevoir Geology and Exploitation (Southwest Petroleum Institute) (2004) A New Relationship of Rock Compressibility with Porosity; SPE 88464

35. D.P. Yale, G.W. Nabor,* and J.A. Russell, Mobil R&D Corp., and H.D. Pham** and Mohamed Yousef, Mobil E&P U.S. Inc. (1993) Application of Variable Formation Compressibility form Improved Reservoir Analysis; SPE 26647

36. Deposition of D. Maclay

37. Deposition of G. Vinson "Pinky"

38. Deposition of J. Loos

39. Deposition of P. Hsieh

40. Deposition of Robert Merrill (Vol 1)

41. Deposition of Robert Merrill (Vol 2)

42. Deposition of S. Chu

43. Deposition Exhibit 8615

44. Deposition Exhibit 8616

45. Deposition Exhibit 8789

46. Expert Report of Kelkar, Mohan and Raghavan, Rajagopal (US)

47. Expert Report of Pooladi-Darvish, Mehran (US)

48. Fortin, J., Guéguen, Y., and Schubnel, A. (2007) Effects of pore collapse and grain crushing on ultrasonic velocities and Vp/Vs. Journal of Geophysical Research, 112, B08207

49. IGS076-001725

50. IMV154-006567

51. IMV365-018434

52. Kelkar, et al., Water-Flooding Incremental Oil Recovery Study in Middle Miocene to Paleocene Reservoirs, Deep-Water Gulf of Mexico, SPE 115669, October 2008

53. Larry H. Britt and Michael B. Smith, NSI Technologies, Inc., Ziad Haddad, Devon Energy Corporation, Jennifer Reese, BP, and Patrick Kelly, BP-Canada Energy Company (2004) Rotary Sidewall Cores - A Cost Effective Means of Determining Young's Modulus; SPE 90861

54. M.H.H. Hettema, C.J. de Pater, (1998) The poromechanical behavior of Felser sandstone: stress-and temperature-dependent; SPE/ISRM 47270

55. P. Romero, J. Lofts, O. Lovera, D. Georgi, Baker Hughes (2011) Flexible Core Sample Acquisition Program for Deepwater, Offshore Brazil; SPE 143800

56. R.M. Ostermeier (1993) SCA Conference Paper number 9315, Some Core Analyses Issues Related to Deepwater Gulf of Mexico Turbidites

57. R.M. Ostermeier (2001) Compaction Effects on Porosity and Permeability: Deepwater Gulf of Mexico Turbidites

58. Recommended Practices for Core Analysis, Recommended Practice 40, Second Edition, February 1998, American Petroleum Institute

59. Teeuw, D. (1971) Prediction of Formation Compaction from Laboratory Compressibility Data; SPE 2973

60. Thomson, L. (1986) Weak elastic anisotropy. Geophysics, 51, 1954-1966

61. Truman, et al., Reserves and Petrophysics, E&P Magazine, June 4, 2004

62. WFT-MDL-00039232

63. WFT-MDL-00039234

64. WFT-MDL-00039235

65. WFT-MDL-00039236

66. WFT-MDL-00039237

67. WFT-MDL-00039238

68. WFT-MDL-00039239

69. WFT-MDL-00039240

70. WFT-MDL-00039241

71. WFT-MDL-00039242

72. WFT-MDL-00039243

73. WFT-MDL-00039244

74. WFT-MDL-00039245

75. WFT-MDL-00039246

76. WFT-MDL-00039247

77. WFT-MDL-00039248

78. WFT-MDL-00039271

79. WFT-MDL-00039272

80. WFT-MDL-00039273

81. WFT-MDL-00039274

82. WFT-MDL-00039275

83. WFT-MDL-00039276

84. WFT-MDL-00039277

85. WFT-MDL-00039278

86. WFT-MDL-00039279

87. WFT-MDL-00039280

88. WFT-MDL-00039281

89. WFT-MDL-00039282

90. WFT-MDL-00039283

91. WFT-MDL-00039284

92. WFT-MDL-00039285

93. WFT-MDL-00039286

94. WFT-MDL-00039287

95. WFT-MDL-00039288

96. WFT-MDL-00039289

97. WFT-MDL-00039290

98. WFT-MDL-00039291

99. WFT-MDL-00039292

100. WFT-MDL-00039293

101. WFT-MDL-00039294

102. WFT-MDL-00039295

103. WFT-MDL-00039296

104. WFT-MDL-00039297

105. WFT-MDL-00039298

106. WFT-MDL-00039299

107. WFT-MDL-00039300

108. WFT-MDL-00039301

109. WFT-MDL-00039302

110. WFT-MDL-00039303

111. WFT-MDL-00039304

112. WFT-MDL-00039305

113. WFT-MDL-00039306

114. WFT-MDL-00039307

115. WFT-MDL-00039308

116. WFT-MDL-00039309

117. WFT-MDL-00039310

118. WFT-MDL-00039311

119. WFT-MDL-00039312

120. WFT-MDL-00039313

121. WFT-MDL-00039314

122. WFT-MDL-00039315

123. WFT-MDL-00039316

124. WFT-MDL-00039317

125. WFT-MDL-00039318

126. WFT-MDL-00039319

127. WFT-MDL-00039320

128. WFT-MDL-00039321

129. WFT-MDL-00039322

130. WFT-MDL-00039323

131. WFT-MDL-00039324

132. WFT-MDL-00039325

133. WFT-MDL-00039326

134. WFT-MDL-00039327

135. WFT-MDL-00039328

136. WFT-MDL-00039329

137. WFT-MDL-00039330

138. WFT-MDL-00039331

139. WFT-MDL-00039332

140. WFT-MDL-00039333

141. WFT-MDL-00039334

142. WFT-MDL-00039335

143. WFT-MDL-00039336

144. WFT-MDL-00039337

145. WFT-MDL-00039338

146. WFT-MDL-00039339

147. WFT-MDL-00039340

148. WFT-MDL-00039341

149. WFT-MDL-00039342

150. WFT-MDL-00039343

151. WFT-MDL-00039344

152. WFT-MDL-00039345

153. WFT-MDL-00039346

154. WFT-MDL-00039346

155. WFT-MDL-00039347

156. WFT-MDL-00039348

157. WFT-MDL-00039349

158. WFT-MDL-00039350

159. WFT-MDL-00039351

160. WFT-MDL-00039352

161. WFT-MDL-00039352

162. WFT-MDL-00039353

163. WFT-MDL-00039354

164. WFT-MDL-00039354

165. WFT-MDL-00039355

166. WFT-MDL-00039356

167. WFT-MDL-00039357

168. WFT-MDL-00039358

169. WFT-MDL-00039359

170. WFT-MDL-00039360

171. WFT-MDL-00039361

172. WFT-MDL-00039362

173. WFT-MDL-00039363

174. WFT-MDL-00039364

175. WFT-MDL-00039365

176. WFT-MDL-00039366

177. WFT-MDL-00039367

178. WFT-MDL-00039368

179. WFT-MDL-00039385

180. WFT-MDL-00039587

181. WFT-MDL-00039615

182. WFT-MDL-00039616

183. WFT-MDL-00039617

184. WFT-MDL-00039618

185. WFT-MDL-00039619

186. WFT-MDL-00039620

187. WFT-MDL-00039621

188. WFT-MDL-00039622

189. WFT-MDL-00039623

190. WFT-MDL-00039624

191. WFT-MDL-00039625

192. WFT-MDL-00039695

193. WFT-MDL-00039696

194. WFT-MDL-00039697

195. WFT-MDL-00039698

196. WFT-MDL-00039699

197. WFT-MDL-00039700

198. WFT-MDL-00039701

199. WFT-MDL-00039702

200. WFT-MDL-00039703

201. WFT-MDL-00039704

202. WFT-MDL-00039705

203. WFT-MDL-00039706

204. WFT-MDL-00039707

205. WFT-MDL-00039708

206. WFT-MDL-00039709

207. WFT-MDL-00039710

208. WFT-MDL-00039711

209. WFT-MDL-00039712

210. WFT-MDL-00039713

211. WFT-MDL-00039714

212. WFT-MDL-00039715

213. WFT-MDL-00039716

214. WFT-MDL-00039717

215. WFT-MDL-00039718

216. WFT-MDL-00039719

217. WFT-MDL-00039720

218. WFT-MDL-00039721

219. WFT-MDL-00039722

220. WFT-MDL-00039723

221. WFT-MDL-00039724

222. WFT-MDL-00039725

223. WFT-MDL-00039726

224. WFT-MDL-00039727

225. WFT-MDL-00039728

226. WFT-MDL-00039729

227. WFT-MDL-00039730

228. WFT-MDL-00039731

229. WFT-MDL-00039732

230. WFT-MDL-00039733

231. WFT-MDL-00039734

232. WFT-MDL-00039735

233. WFT-MDL-00039736

234. WFT-MDL-00039737

235. WFT-MDL-00039738

236. WFT-MDL-00039739

237. WFT-MDL-00039740

238. WFT-MDL-00039741

239. WFT-MDL-00039742

240. WFT-MDL-00039743

241. WFT-MDL-00039744

242. WFT-MDL-00039745

243. WFT-MDL-00039746

244. WFT-MDL-00039747

245. WFT-MDL-00039748

246. WFT-MDL-00039838

247. WFT-MDL-00039839

248. WFT-MDL-00039840

249. WFT-MDL-00039841

250. WFT-MDL-00054278

251. WFT-MDL-00054279

252. WFT-MDL-00054280

253. WFT-MDL-00054281

254. WFT-MDL-00054282

255. WFT-MDL-00054283

256. WFT-MDL-00082902

257. WFT-MDL-00082904

258. WFT-MDL-00129170

259. WFT-MDL-00129171

260. WFT-MDL-00129617

261. WFT-MDL-00130933

262. WFT-MDL-00131648

263. Zimmerman, R.W., Somerton, W.H., and King, M.S. (1986) Compressibility of porous rocks, Journal of Geophysical Research, 91, 12765-12778