From: Hedeff I Essaid <hiessaid@usgs.gov>
Sent: Thursday, October 14, 2010 5:21 PM
To: pahsieh@usgs.gov
Subject: Colleague Review
Attachments: Attachment

Hi Paul,

I enjoyed reading your report and am impressed with how much you were able to learn from the limited information you were provided with.

I have made minor comments/suggestions that mainly relate to making the report easily accessible to the general reader.

Hedeff

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Computer simulation of Reservoir Depletion and Oil Flow from the Macondo Well Following the Deepwater Horizon Incident

By Paul A. Hsieh

Open-File Report 2010–xxxx

U.S. Department of the Interior
U.S. Geological Survey

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Figure 5. Simulated reservoir pressure at the well face. The origin of the time axis corresponds to April 20, 2010, the date of the Deepwater Horizon blowout.

Figure 6. Simulated volumetric flow rate of oil in stock tank barrels per day. The origin of the time axis corresponds to April 20, 2010, the date of the Deepwater Horizon blowout.

Computer Simulation of Reservoir Depletion and Oil Flow from the Macondo Well Following the Deepwater Horizon Incident

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By Paul A. Hsieh

Abstract

This report describes the application of a computer model to simulate reservoir depletion and oil flow from the Macondo well following the Deepwater Horizon blowout incident. Reservoir and fluid properties used in the model are based on: (1) information provided by BP personnel during meetings in Houston, Texas, and (2) calibration by history matching to wellhead shut-in pressures measured during the Well Integrity Test. In the model simulation of the 86-day period from the blowout to shut in, the simulated reservoir pressure at the well face declines from the initial reservoir pressure of 11,850 psi to 9,400 psi. After shut in, the simulated reservoir pressure recovers to 10,300 psi. The pressure does not recover back to the initial pressure due to reservoir depletion caused by 86 days of oil discharge. The simulated oil flow rate declines from 61,300 stock tank barrels per day just after the Deepwater Horizon blowout to 51,000 stock tank barrels per day just prior to shut in. The simulated total volume of oil discharge is 4.76 million stock tank barrels. Analysis of the predictive uncertainty of the reservoir model suggests that the 95-percent prediction intervals of the simulated flow rates and total discharge are relatively narrow—the upper or lower limits are no more than a few percent higher or lower than the corresponding simulated value. However, these predictions intervals do not fully characterize the uncertainty in the simulated values. Uncertainty in the assumed reservoir and fluid properties used in the model would widen the prediction intervals.

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Background

The computer simulation described in this report was undertaken to supplement the work of the Flow Rate Technical Group, a group of scientists and engineers led by U.S. Geological Survey Director Marcia McNutt to estimate the flow of oil from the Macondo well following the Deepwater Horizon blowout incident. Much of the work of the Flow Rate Technical Group was carried out prior to July 15, 2010, the date when the Macondo well was shut in to begin the Well Integrity Test. The computer simulation described in this report was carried out to analyze the pressure data obtained during the Well Integrity Test in order to gain additional knowledge of the Macondo well and the oil reservoir. A simulation result of particular interest is the assessment of reservoir depletion resulting from oil flow during the 86 days from blowout to shut in. The computer simulation also provided estimates of oil flow rates, which can be used for comparison with the estimates made by the Flow Rate Technical Group.

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Comment [HE1]: Were these comparisons made? Are they in another document? Should it be cited? This sentence makes the reader think that the comparison is going to be made.

A significant amount of information (for example, reservoir and fluid properties) used in the development of the reservoir model described in this report was provided by BP personnel at meetings in Houston, Texas, during the period from late June to early August, 2010. Table 1 shows reservoir and fluid properties that are considered best estimates at the time of the oil spill response. Much of the information is unpublished, and therefore citations could not be provided in this report. Instead, this report focuses on documenting the procedure for developing the reservoir model.

Reservoir Model

Reservoir Geometry and Conditions

The Macondo well produces oil from an oil reservoir known as M56. According to drilling logs, the M56 oil reservoir consists of three oil-producing sand layers. The top of the reservoir is penetrated by the Macondo well at a depth of approximately 18,000 ft TVDSS (True Vertical Depth Sub Sea). The combined thickness of the three oil-producing sand layers is approximately 90 ft. Analysis of seismic data suggests that these oil-producing sands are submarine channel fills, with a longitudinal axis approximately in a northwest-southeast orientation. The initial reservoir pressure was 11,850 psi. Reservoir temperature was

Comment [HE2]: Was this a value provided by BP? Was it based on measurement?

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approximately 240° F. As the bubble point of the oil at this temperature is approximately 6,500 psi, the reservoir is believed to be under single-phase (liquid oil) condition. The estimated volume of "original oil in place" is 1.1×10^8 stock tank barrels. The bulk volume of reservoir containing the oil can be estimated by

Comment [HE3]: From BP?

$$V_b = \frac{V_o B}{\phi(1 - S_w)}, \tag{1}$$

where

- V_b is the bulk volume of reservoir containing the oil,
- V_o is the volume of original oil in place,
- B is the formation volume factor,
- ϕ is porosity, and
- S_w is water saturation.

Comment [HE4]: Include units in equation definitions?

Using reservoir properties given in Table 1, the bulk volume of reservoir containing the oil is computed to be 7.68×10^9 ft^3.

In the model, the oil reservoir is represented by a long, narrow channel having a rectangular cross section (Figure 1). The vertical thickness (b) of the channel is 90 ft. The horizontal length (L) and width (W) are initially unknown and are estimated by history matching. However, because $L \times W \times b$ must equal V_b, L and W are related by

$$L \times W = \frac{V_b}{b} = \frac{7.68 \times 10^9 \text{ ft}^3}{90 \text{ ft}} = 8.53 \times 10^7 \text{ ft}^2 \tag{2}$$

The reservoir is assumed to be a closed system. In other words, all six faces of the channel are impermeable boundaries. Within the reservoir, the Macondo well location is defined by the coordinates (x_w, y_w), which are initially unknown and are estimated during history matching of the Well Integrity Test.

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Mathematical Formulation

The equation of oil flow in the reservoir is given by (Matthews and Russell, 1967)

$$\frac{\partial^2 p}{\partial x^2}+\frac{\partial^2 p}{\partial y^2}=\frac{\phi\mu c}{k}\frac{\partial p}{\partial t}, \quad (3)$$

where

p is pressure,

c is compressibility,

k is permeability,

μ is oil viscosity,

x, y are Cartesian coordinates in the horizontal plane, and

t is time.

Comment [HE5]: Units?

In applying equation 3 to the reservoir, the following conditions are assumed:

1. Flow of oil is under single-phase and isothermal conditions,
2. Reservoir properties (permeability, porosity, and compressibility) are homogeneous,
3. Permeability and viscosity are independent of pressure, and
4. Permeability is isotropic.

The compressibility is computed as (Matthews and Russell, 1967)

$$c=(1-S_w)c_o+S_w c_w+c_f, \quad (4)$$

where

c_o is oil compressibility,

c_w is water compressibility, and

c_f is effective formation (or pore) compressibility.

Comment [HE6]: Units?

Except for permeability, values of reservoir and fluid properties used in the reservoir model are given in Table 1. Permeability is estimated from history matching.

The volumetric flow rate of oil from the reservoir through the Macondo well and exiting the blowout preventer is modeled by the equation (see Figure 2)

Comment [HE7]: Is this also from Matthews and Russell? Give citation.

$$Q^2 = C(p_w - \Delta - p_e), \tag{5}$$

where

Q is volumetric flow rate of oil at reservoir conditions,

C is a coefficient of pressure loss through the well,

p_w is the reservoir pressure at the well face,

Δ is the pressure correction to account for the elevation difference between reservoir and the exit point at the blowout preventer, and

Comment [HE8]: Units?

p_e is the ambient pressure at the exit point of the blowout preventer.

The pressure correction Δ is computed by (see Figure 2)

$$\Delta = G_o(d_r - d_e), \tag{6}$$

where

G_o is the oil pressure gradient in the well,

d_r is the depth of the reservoir, and

d_e is the depth of the exit point at the blowout preventer.

Comment [HE9]: Is this a value recommended by BP? Or some other information?

Comment [HE10]: Was this measured or estimated using depth and density of sea water?

Comment [HE11]: I suggest moving this explanation up to where the equatin is introduced.

For the Macondo well flow calculation, G_o is taken to be 0.25 psi/ft, d_r is 18,000 ft TVDSS, and d_e is 5,000 ft TVDSS. Therefore, Δ is computed to be 3,250 psi. The ambient pressure at the exit point of the blowout preventer, p_e, is 2,190 psi. The volumetric flow rate of oil at surface (stock tank) conditions is computed by dividing Q by the formation volume factor B.

The Q^2 term in Equation 5 is based on the assumption that flow is turbulent in the well. The value of the coefficient C is initially unknown and is estimated by history matching. In the reservoir simulation, C is kept constant in time for the entire period of well flow. This assumes that the changes in outlet configuration, such as cutting of the riser pipe, do not significantly impact the oil flow rate.

MODFLOW Implementation

The U.S. Geological Survey model known as MODFLOW-2000 (Harbaugh and others, 2000) is used to simulate oil flow in the M56 oil reservoir. Although MODFLOW-2000 is originally designed to simulate the flow of groundwater in aquifers, it can be readily adapted for simulating flow of oil in reservoirs under single-phase and isothermal conditions. The fluid flow equation solved by MODFLOW-2000 is analogous to Equation 3, and can be written as

$$\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial y^2}=\frac{S_s}{K}\frac{\partial h}{\partial t}, \tag{7}$$

where

h is hydraulic head,

K is hydraulic conductivity, and

S_s is specific storage.

For simulating oil flow, the quantities h, K, and S_s are computed as

$$h=\frac{p}{\rho_o g}+z, \tag{8}$$

$$K=\frac{\rho_o g k}{\mu}, \tag{9}$$

$$S_s=\rho_o g\phi c, \tag{10}$$

where

ρ_o is oil density,

z is vertical elevation above a given datum, and

g is gravitational acceleration.

A modified version of the General-Head Boundary Package is used to simulate flow through the Macondo well, as expressed by Equation 5. In its original version, the General-Head Boundary Package can be used to implement Equation 5 if the exponent of the Q term were 1 instead of 2.

Deleted: simulation

Comment [HE12]: I think that it would be worthwhile to show the equation for the general head boundary (for the general reader) so that one can see the correspondence between it and equation 5.

To implement the Q^2 term, the Fortran source code of the General-Head Boundary Package is modified and the program recompiled.

Figure 3 is a map view showing an example finite-difference grid of the oil reservoir, which is represented by 90 ft. thick single model layer. The cell containing the Macondo well has a horizontal dimension of 1 ft by 1 ft. The cell size increases away from the well to a maximum size of 100 ft. The simulation time step is 0.2 day.

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Comment [HE13]: I think that the model setup needs to be explained a little further. Explain that no flow boundaries were used on the edges, explain simulation period (i.e. that it is more than just the shut-in time period), explain how the well boundary condition was adjusted when going from the flowing period to the shut-in period.

History Matching

The parameter estimation program PEST version 10 (Doherty, 2004) is used to perform history matching—the adjustment of model parameters so that simulated pressures match measured pressures. PEST implements a nonlinear least-squares regression method to estimate model parameters by minimizing the sum of squares of the differences between measured and simulated pressures:

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$$\Phi = \sum_{i=1}^{N}(p_i^{mea} - p_i^{sim})^2 \quad (11)$$

Where

N is the number of measurements,

p_i^{mea} is the the i^{th} measured pressure, and

p_i^{sim} is the i^{th} simulated pressure.

PEST uses the Gauss-Marquardt-Levenberg method to minimize Φ. Details of this method are given in the PEST user's manual (Doherty, 2004).

The pressure data used for history matching were measured during the Well Integrity Test, which began on July 15, 2010. At 2:30 pm Central Daylight Time, the final turn on the choke was closed and the Macondo well was shut in. Wellhead pressures were measured by two pressure gages installed in the sealing cap. For history matching, wellhead pressures measured by the gage known as "PT-3K-2" are used. The simulated wellhead pressure is calculated by subtracting the Δ value of 3.198 psi (see Equation 6) from the simulated reservoir pressure at the well face to adjust for the 13,000 ft elevation difference between the M56 reservoir and the pressure gage.

Comment [HE14]: Is this simply the oil density times g times delta z?

Figure 4 is a Horner plot showing the measured and simulated wellhead pressures during the Well Integrity Test. The horizontal axis of the Horner plot shows the quantity $(t_p + \Delta t)/\Delta t$, where t_p is the period of oil flow (86 days), and Δt is the time since shut in. Note that on the horizontal axis, time increases to the left. The left-most pressure measurement in the plot was taken on August 3, 2010, which is 19 days after shut in. Figure 4 shows that the simulated pressures closely match the measured pressured. The model parameter values estimated by history matching are given in Table 2.

Simulation Results

Reservoir Depletion

Figure 5 shows the simulated reservoir pressure at the Macondo well face. The origin of the time axis corresponds to April 20, 2010, the date of the Deepwater Horizon blowout. The initial reservoir pressure was 11,850 psi. Immediately after the blowout, the simulated pressure drops rapidly to approximately 11,000 psi and then follows a steady decline to 9,400 psi on day 86, just prior to shut in. After shut in, the simulated pressure recovers and eventually stabilizes at 10,300 psi. The pressure does not recover back to the initial pressure due to reservoir depletion from 86 days of oil discharge.

Comment [HE15]: Was this measured or estimated by BP?

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Oil Flow Rate

Figure 6 shows the simulated volumetric flow rate of oil for surface conditions (expressed in stock tank barrels per day). The simulated initial volumetric flow rate of oil is 61,300 stock tank barrels per day. As the reservoir depletes, the flow rate decreases to 51,000 stock tank barrels per day on day 86, just prior to shut in. The simulated total volume of oil discharge over the 86-day period from blowout to shut in is 4.76 million stock tank barrels.

Comment [HE16]: I think that you should also express this as a fraction of the estimate of original oil in place – I calculated it to be about 4% of the original oil.

Uncertainty Analysis

After history matching, the program PEST is run in "predictive analysis mode" to assess the predictive uncertainty of the reservoir model (See Doherty, 2004, Chapter 6). In this context, a "prediction" is simply a model simulated quantity that is not measured—there is no implication that the simulated quantity is to occur in the future. Three simulated quantities are of particular interest: (1) the initial oil flow rate, just after the blowout, (2) the final oil flow rate, just before shut in, and (3) the total volume of oil discharged. Table 3 gives the predictive uncertainty of these simulated quantities in terms of 95% prediction intervals. Note that all three intervals are relatively narrow—the upper or lower limits are no more than a few percent higher or lower than the corresponding simulated value. The narrow intervals are largely due to the close match between simulated and observed pressures, and the low degree of nonuniqueness in the estimated parameters.

It is important to note that the prediction intervals given in Table 3 do not fully characterize the uncertainty in the simulated values. In calculating these intervals, it is assumed that the reservoir and fluid properties given in Table 1 are known. However, quantities such as original oil in place are, in fact, best estimates and are subject to uncertainty. Although such uncertainties cannot be quantitatively assessed in the present study, they would widen the prediction intervals of the simulated quantities in Table 3.

Conclusions

The reservoir model presented in this report simulates oil discharge from the Macondo well following the Deepwater Horizon blowout and pressure recovery after the well was shut in. During the 86-day period of oil discharge, the simulated reservoir pressure at the well face declines from the initial reservoir pressure of 11,850 psi to 9,400 psi. After shut in, the simulated reservoir pressure recovers to 10,300 psi. The pressure does not recover back to the initial pressure due to reservoir depletion from the oil discharge. The simulated oil flow rate declines from 61,300 stock tank barrels per day just after the Deepwater Horizon blowout to 51,000 stock tank barrels per day just prior to shut in. The simulated total volume of oil discharge is 4.76 million stock tank barrels. Analysis of the predictive uncertainty of the reservoir model suggests that the 95-percent prediction intervals of the simulated flow rates and total discharge are relatively narrow—the upper or lower limits are no more than a few percent higher or lower than the corresponding simulated value. However, these predictions intervals do not fully characterize the uncertainty in the simulated values. Uncertainty in the assumed reservoir and fluid properties used in the model would widen the prediction intervals.

References Cited

Doherty, John, 2004, PEST model-independent parameter estimation user manual, 5th Edition: Watermark Numerical Computing, variously paged, accessed October 5, 2010, at http://www.pesthomepage.org/Downloads.php.

Harbaugh, A.W., Banta, E.R., Hill, M.C., and McDonald, M.G., 2000, MODFLOW-2000, the U.S. Geological Survey modular ground-water model—User guide to modularization concepts and the ground-water flow process: U.S. Geological Survey Open-File Report 00-92, 121 p., accessed October 5, 2010, at http://water.usgs.gov/nrp/gwsoftware/modflow2000/ofr00-92.pdf.

Matthews, C.S., and Russell, D.G., 1967, Pressure Buildup and Flow Tests in Wells: New York, Society of Petroleum Engineers of AIME.

Table 1. Reservoir and fluid properties used in reservoir model. Values are given for reservoir conditions.

Comment [HE17]: Were all of these values from BP? If not, it would be helpful to know which were from BP

Reservoir or Fluid Property	Value Used in Reservoir Model
Original oil in place	1.1×10^{-8} stock tank barrels
Formation volume factor, B	2.35
Porosity, ϕ	0.21
Effective formation (or pore) compressibility, c_f	1.2×10^{-5} psi^{-1}
Oil viscosity, μ	0.168 cp
Oil compressibility, c_o	1.46×10^{-5} psi^{-1}
Oil density, ρ_o	35.46 lb/ft^3
Water saturation, S_w	0.1
Water compressibility, c_w	3.0×10^{-6} psi^{-1}

Table 2. Values of model parameters estimated from history matching. See Figure 1 for definition of L, W, x_w, and y_w.

Model Parameter	Estimated Value from History Matching
Horizontal length of reservoir, L	22,270 ft
Horizontal width of reservoir, W	3,830 ft
X-coordinate of Macondo well, x_w	3,100 ft
Y-coordinate of Macondo well, y_w	2,700 ft
permeability, k	570 millidarcy
Coefficient of pressure loss in well, C	3.56×10^6 (barrels/day)^2/psi

Table 3. Simulated values and 95-precent prediction intervals computed by running PEST in predictive analysis mode.

Simulated Quantity	Simulated Value	95% Prediction Interval Min	95% Prediction Interval Max
Initial oil flow rate (stock tank barrels/day)	61,300	60,500	61,900
Final oil flow rate (stock tank barrels/day)	51,000	50,300	51,600
Total volume of oil discharged (stock tank barrels)	4.76×10^6	4.69×10^6	4.81×10^6

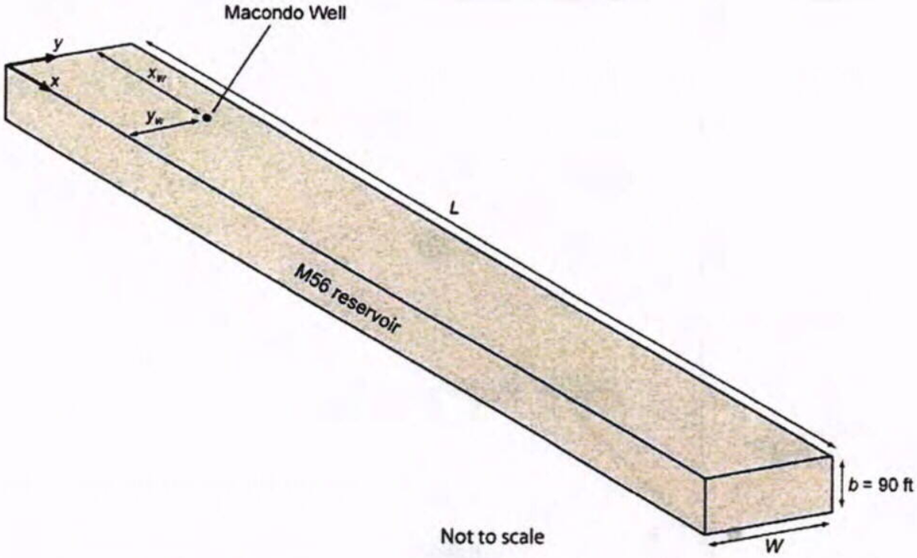


Figure 1. Oblique view of the M56 reservoir.

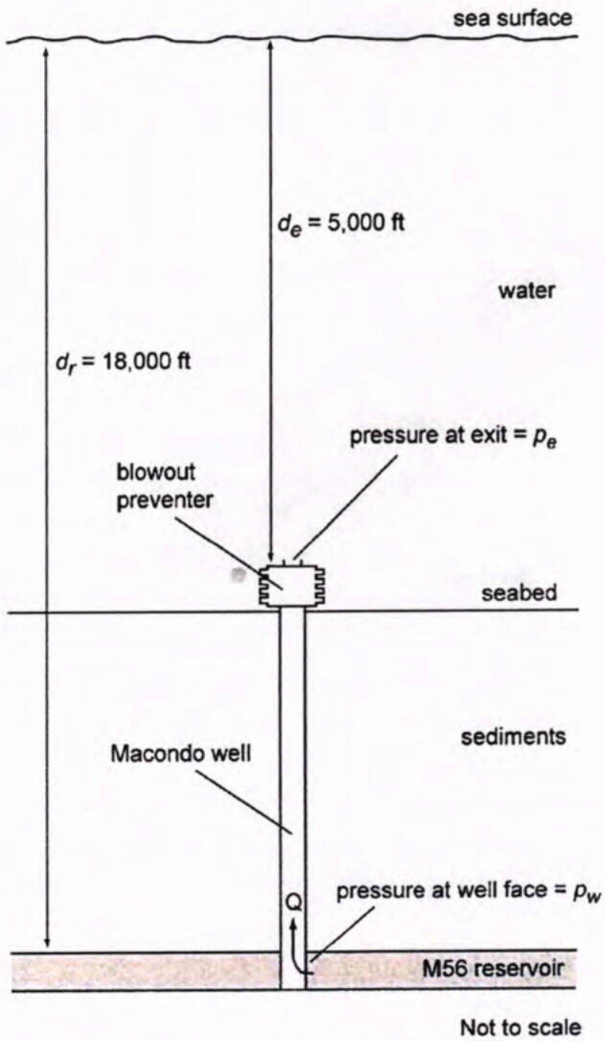


Figure 2. Schematic vertical section showing flow of oil from M56 reservoir through the Macondo well and exiting at the top of the blowout preventer.

Comment [HE18]: I would have liked to see this schematic also relate to the model boundary conditions. Maybe you could show the correspondence with the simulated value (=well face) and the well head value (rather than just the pressure at the exit)

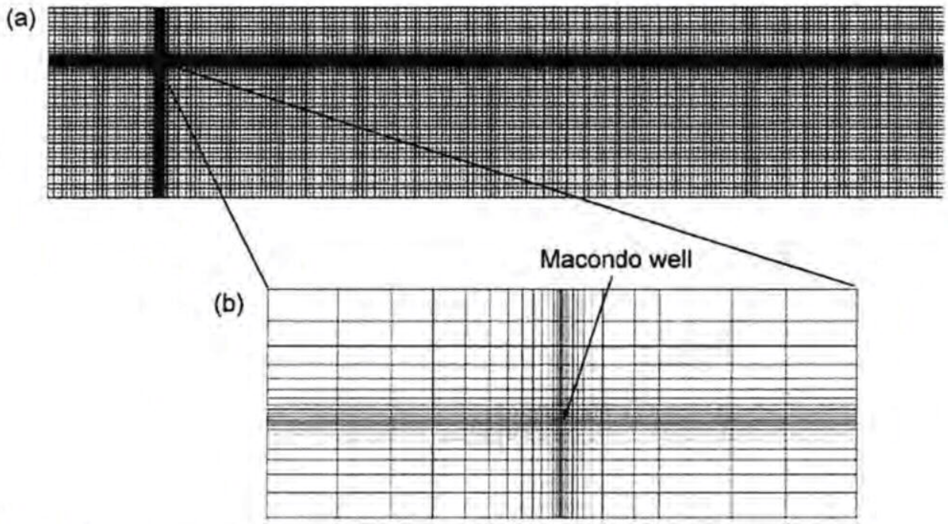


Figure 3. Map view of an example finite-difference grid of the oil reservoir. (a) Entire grid. (b) Detailed view of a small portion of the grid in the vicinity of the Macondo well.

Comment [HE19]: Was the gridding changed during the history matching as x and y changed?

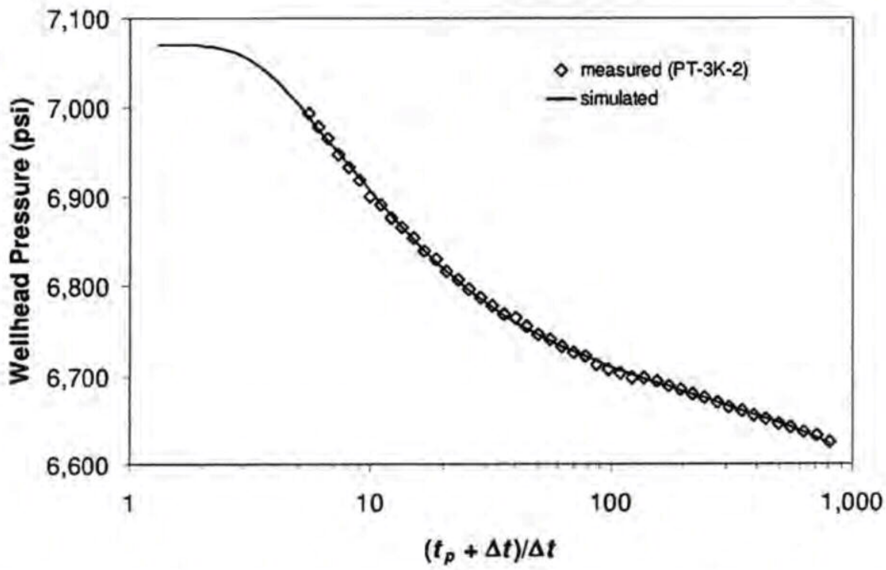


Figure 4. Horner plot of simulated and measured wellhead pressure during Well Integrity Test. t_p is the period of oil flow, which is 86 days. Δt is time since shut in. Note that time increases to the left on the horizontal axis.

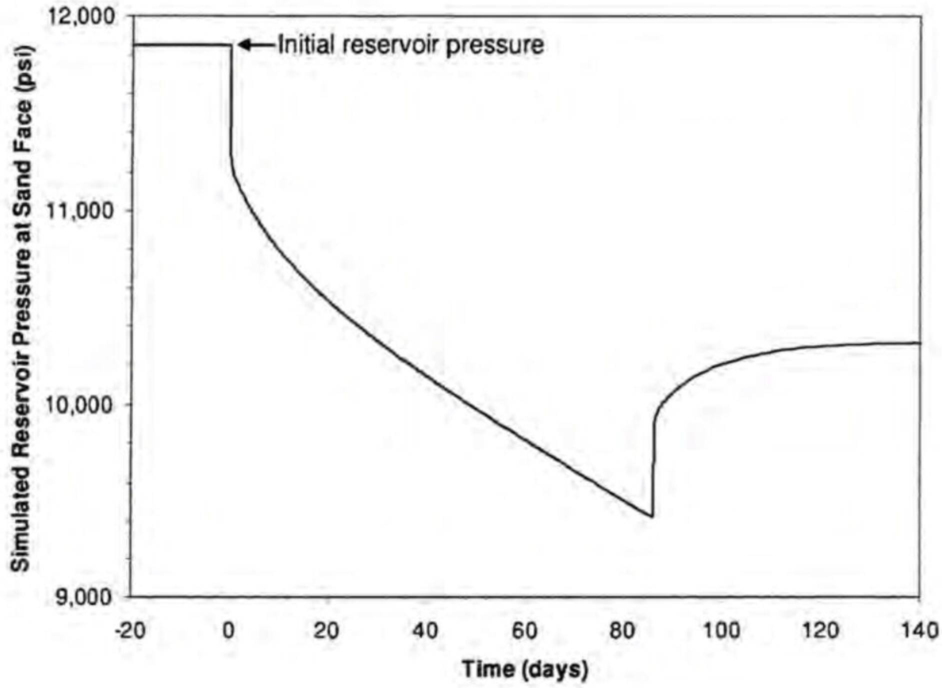


Figure 5. Simulated reservoir pressure at the well face. The origin of the time axis (t = 0) corresponds to April 20, 2010, the date of the Deepwater Horizon blowout.

Comment [HE20]: Y axis label says 'sand face'

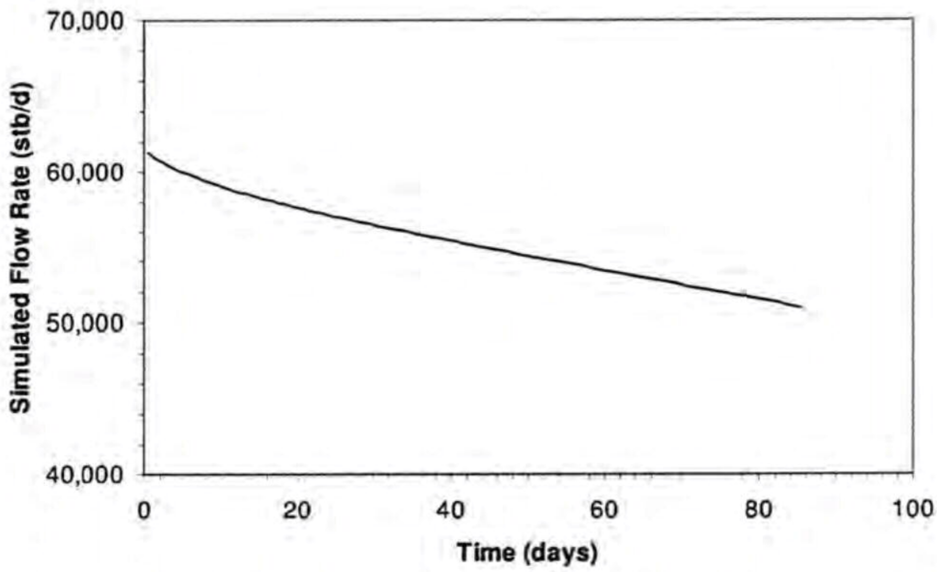
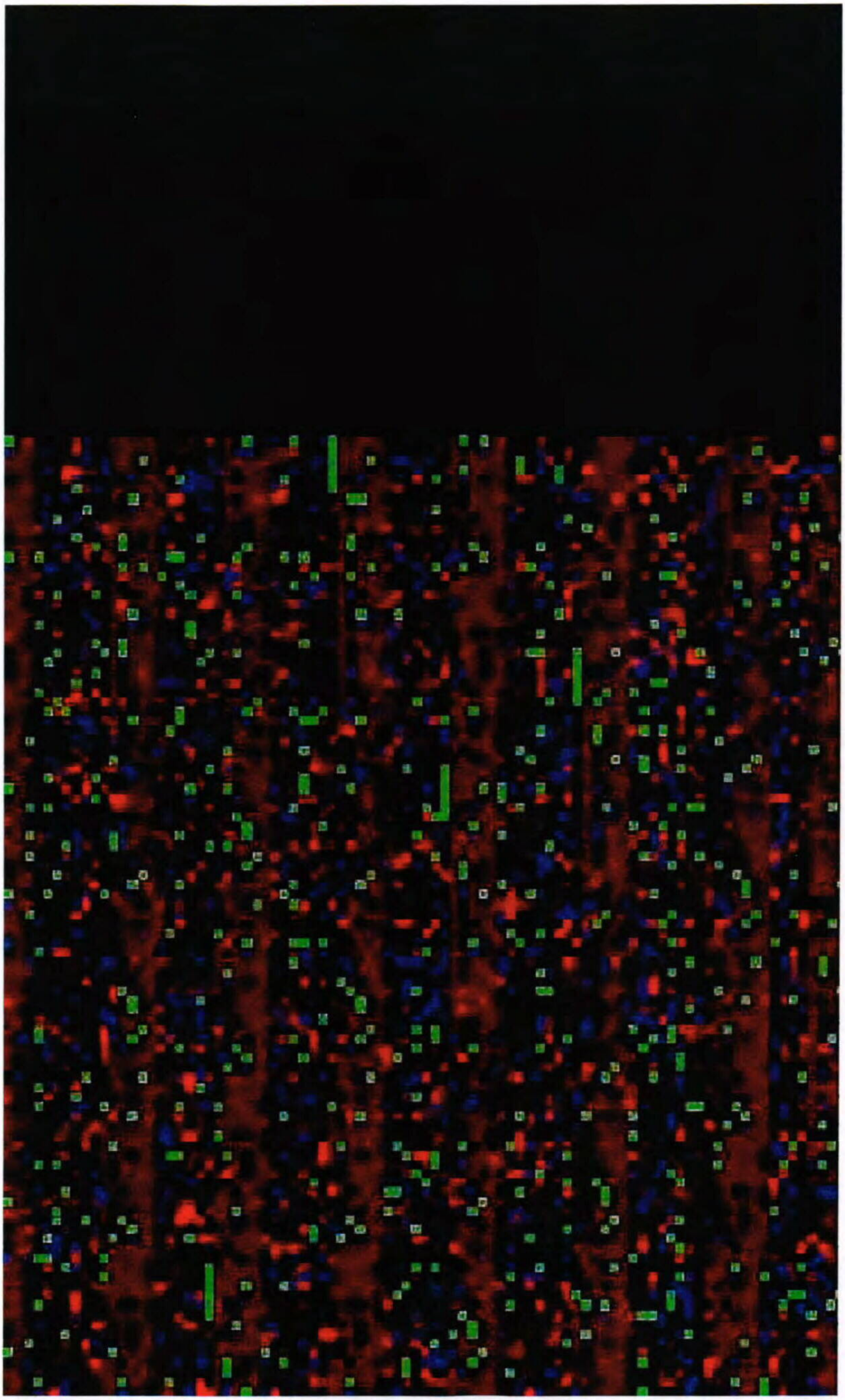


Figure 6. Simulated volumetric flow rate of oil in stock tank barrels per day. The origin of the time axis ($t = 0$) corresponds to April 20, 2010, the date of the Deepwater Horizon blowout.



From: Barbara A Bekins <babekins@usgs.gov>
Sent: Friday, October 15, 2010 12:56 AM
To: pahsieh@usgs.gov
Subject: draft report comments
Attachments: Attachment

Hi Paul,

The report is very impressive. Thanks for the opportunity to review it. I am attaching a version with a few comments.

--Barbara

==

Barbara Bekins, Ph.D.

Research Hydrologist

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Ph: (650) 329-4691; Fax: (650) 329-4463

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A significant amount of information (for example, reservoir and fluid properties) used in the development of the reservoir model described in this report was provided by BP personnel at meetings in Houston, Texas, during the period from late June to early August, 2010. Table 1 shows reservoir and fluid properties that are considered best estimates at the time of the oil spill response. Much of the information is unpublished, and therefore citations could not be provided in this report. Instead, this report focuses on documenting the procedure for developing the reservoir model.

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Comment [b1]: Consider providing a definition for this

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$$L \times W = \frac{V_b}{b} = \frac{7.68 \times 10^9 \text{ ft}^3}{90 \text{ ft}} = 8.53 \times 10^7 \text{ ft}^2 \tag{2}$$

The reservoir is assumed to be a closed system. In other words, all six faces of the channel are impermeable boundaries. Within the reservoir, the Macondo well location is defined by the coordinates (x_w, y_w), which are initially unknown and are estimated by history matching.

Mathematical Formulation

The equation of oil flow in the reservoir is given by (Matthews and Russell, 1967)

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = \frac{\phi \mu c}{k} \frac{\partial p}{\partial t}, \qquad (3)$$

where

p is pressure,

c is compressibility,

k is permeability,

μ is oil viscosity,

x, y are Cartesian coordinates in the horizontal plane, and

t is time.

In applying equation 3 to the reservoir, the following conditions are assumed:

1. Flow of oil is under single-phase and isothermal conditions,
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3. Permeability and viscosity are independent of pressure, and
4. Permeability is isotropic.

The compressibility is computed as (Matthews and Russell, 1967)

$$c = (1 - S_w)c_o + S_w c_w + c_f, \qquad (4)$$

where

c_o is oil compressibility,

c_w is water compressibility, and

c_f is effective formation (or pore) compressibility.

Except for permeability, values of reservoir and fluid properties used in the reservoir model are given in Table 1. Permeability is estimated from history matching.

The volumetric flow rate of oil from the reservoir through the Macondo well and exiting the blowout preventer is modeled by the equation (see Figure 2)

$$Q^2 = C(p_w - \Delta - p_e), \quad (5)$$

where

Q is volumetric flow rate of oil at reservoir conditions,

C is a coefficient of pressure loss in the well,

p_w is the reservoir pressure at the well face,

Δ is the pressure correction to account for the elevation difference between reservoir and the exit point at the blowout preventer, and

p_e is the ambient pressure at the exit point of the blowout preventer.

Comment [b2]: Use same wording as Table 1

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The pressure correction Δ is computed by (see Figure 2)

$$\Delta = G_o(d_r - d_e), \quad (6)$$

where

G_o is the oil pressure gradient in the well,

d_r is the depth of the reservoir, and

d_e is the depth of the exit point at the blowout preventer.

For the Macondo well flow calculation, G_o is taken to be 0.25 psi/ft, d_r is 18,000 ft TVDSS, and d_e is 5,000 ft TVDSS. Therefore, Δ is computed to be 3,250 psi. The ambient pressure at the exit point of the blowout preventer, p_e, is 2,190 psi. The volumetric flow rate of oil at surface (stock tank) conditions is computed by dividing Q by the formation volume factor B.

Comment [b3]: Consider adding these to Table 1

The Q^2 term in Equation 5 is based on the assumption that flow is turbulent in the well. The value of the coefficient C is initially unknown and is estimated by history matching. In the reservoir simulation, C is kept constant in time for the entire period of well flow. This assumes that the changes in outlet configuration, such as cutting of the riser pipe, do not significantly impact the oil flow rate.

MODFLOW Implementation

The U.S. Geological Survey model known as MODFLOW-2000 (Harbaugh and others, 2000) is used to simulate oil flow in the M56 oil reservoir. Although MODFLOW-2000 is originally designed to simulate the flow of groundwater in aquifers, it can be readily adapted for simulating flow of oil in reservoirs under single-phase and isothermal conditions. The fluid flow equation solved by MODFLOW-2000 is analogous to Equation 3, and can be written as

$$\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial y^2}=\frac{S_s}{K}\frac{\partial h}{\partial t}, \tag{7}$$

where

h is hydraulic head,

K is hydraulic conductivity, and

S_s is specific storage.

For simulating oil flow, the quantities h, K, and S_s are computed as

$$h=\frac{p}{\rho_o g}+z, \tag{8}$$

$$K=\frac{\rho_o g k}{\mu}, \tag{9}$$

$$S_s=\rho_o g \phi c, \tag{10}$$

where

ρ_o is oil density,

z is vertical elevation above a given datum, and

g is gravitational acceleration.

A modified version of the General-Head Boundary Package is used to simulation flow through the Macondo well, as expressed by Equation 5. In its original version, the General-Head Boundary Package can be used to implement Equation 5 if the exponent of the Q term were 1 instead of 2. To implement the Q^2 term, the Fortran source code of the General-Head Boundary Package was modified and the program recompiled.

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Figure 3 is a map view showing an example finite-difference grid of the oil reservoir, which is represented by one model layer. The cell containing the Macondo well has a horizontal dimension of 1 ft by 1 ft. The cell size increases away from the well to a maximum size of 100 ft. The simulation time step is 0.2 day.

History Matching

The parameter estimation program PEST version 10 (Doherty, 2004) is used to perform history matching—the adjustment of model parameters so that simulated pressures match measured pressures. PEST implements a nonlinear least-squares regression method to estimate model parameters by minimizing the sum of squares of the differences between measured and simulated pressures:

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$$\Phi = \sum_{i=1}^{N}(p_i^{mea} - p_i^{sim})^2 \qquad (11)$$

Where

N is the number of measurements,

p_i^{mea} is the the i^{th} measured pressure, and

p_i^{sim} is the i^{th} simulated pressure.

PEST uses the Gauss-Marquardt-Levenberg method to minimize Φ. Details of this method are given in the PEST user's manual (Doherty, 2004).

The pressure data used for history matching were measured during the Well Integrity Test, which began on July 15, 2010. At 2:30 pm Central Daylight Time, the final turn on the choke was closed and the Macondo well was shut in. Wellhead pressures were measured by two pressure gages installed in the sealing cap. For history matching, wellhead pressures measured by the gage known as "PT-3K-2" are used. The simulated wellhead pressure is calculated by subtracting the Δ value of 3,198 psi (see Equation 6) from the simulated reservoir pressure at the well face to adjust for the 13,000 ft elevation difference between the M56 reservoir and the pressure gage.

Comment [b4]: When did simulation begin?

Figure 4 is a Horner plot showing the measured and simulated wellhead pressures during the Well Integrity Test. The horizontal axis of the Horner plot shows the quantity $(t_p + \Delta t)/\Delta t$, where t_p is the period of oil flow (86 days), and Δt is the time since shut in. Note that on the horizontal axis, time increases to the left. The left-most pressure measurement in the plot was taken on August 3, 2010, which is 19 days after shut in. Figure 4 shows that the simulated pressures closely match the measured pressures. The model parameter values estimated by history matching are given in Table 2.

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Simulation Results

Reservoir Depletion

Figure 5 shows the simulated reservoir pressure at the Macondo well face. The origin of the time axis corresponds to April 20, 2010, the date of the Deepwater Horizon blowout. The initial reservoir pressure is 11,850 psi. Immediately after the blowout, the simulated pressure drops rapidly to approximately 11,000 psi and then follows a steady decline to 9,400 psi on day 86, just prior to shut in. After shut in, the simulated pressure recovers and eventually stabilizes at 10,300 psi. The pressure does not recover back to the initial pressure due to reservoir depletion from 86 days of oil discharge.

Oil Flow Rate

Figure 6 shows the simulated volumetric flow rate of oil for surface conditions (expressed in stock tank barrels per day). The simulated initial volumetric flow rate of oil is 61,300 stock tank barrels per day. As the reservoir depletes, the flow rate decreases to 51,000 stock tank barrels per day on day 86, just prior to shut in. The simulated total volume of oil discharge over the 86-day period from blowout to shut in is 4.76 million stock tank barrels.

Uncertainty Analysis

After history matching, the program PEST is run in "predictive analysis mode" to assess the predictive uncertainty of the reservoir model (See Doherty, 2004, Chapter 6). In this context, a "prediction" is simply a model simulated quantity that is not measured—there is no implication that the simulated quantity is to occur in the future. Three simulated quantities are of particular interest: (1) the initial oil flow rate, just after the blowout, (2) the final oil flow rate, just before shut in, and (3) the total volume of oil discharged. Table 3 gives the predictive uncertainty of these simulated quantities in terms of 95% prediction intervals. Note that all three intervals are relatively narrow—the upper or lower limits are no more than a few percent higher or lower than the corresponding simulated value. The narrow intervals are largely due to the close match between simulated and observed pressures, and the low degree of nonuniqueness in the estimated parameters.

It is important to note that the prediction intervals given in Table 3 do not fully characterize the uncertainty in the simulated values. In calculating these intervals, it is assumed that the reservoir and fluid properties given in Table 1 are known. However, quantities such as original oil in place are, in fact, best estimates and are subject to uncertainty. Although such uncertainties cannot be quantitatively assessed in the present study, they would widen the prediction intervals of the simulated quantities in Table 3.

Conclusions

The reservoir model presented in this report simulates oil discharge from the Macondo well following the Deepwater Horizon blowout and pressure recovery after the well was shut in. During the 86-day period of oil discharge, the simulated reservoir pressure at the well face declines from the initial reservoir pressure of 11,850 psi to 9,400 psi. After shut in, the simulated reservoir pressure recovers to 10,300 psi. The pressure does not recover back to the initial pressure due to reservoir depletion from the oil discharge. The simulated oil flow rate declines from 61,300 stock tank barrels per day just after the Deepwater Horizon blowout to 51,000 stock tank barrels per day just prior to shut in. The simulated total volume of oil discharge is 4.76 million stock tank barrels. Analysis of the predictive uncertainty of the reservoir model suggests that the 95-percent prediction intervals of the simulated flow rates and total discharge are relatively narrow—the upper or lower limits are no more than a few percent higher or lower than the corresponding simulated value. However, these predictions intervals do not fully characterize the uncertainty in the simulated values. Uncertainty in the assumed reservoir and fluid properties used in the model would widen the prediction intervals.

References Cited

Doherty, John, 2004, PEST model-independent parameter estimation user manual, 5th Edition: Watermark Numerical Computing, variously paged, accessed October 5, 2010, at http://www.pesthomepage.org/Downloads.php.

Harbaugh, A.W., Banta, E.R., Hill, M.C., and McDonald, M.G., 2000, MODFLOW-2000, the U.S. Geological Survey modular ground-water model—User guide to modularization concepts and the ground-water flow process: U.S. Geological Survey Open-File Report 00-92, 121 p., accessed October 5, 2010, at http://water.usgs.gov/nrp/gwsoftware/modflow2000/ofr00-92.pdf.

Matthews, C.S., and Russell, D.G., 1967, Pressure Buildup and Flow Test in Wells: New York, Society of Petroleum Engineers of AIME.

Comment [b5]: Number of pages?

Table 1. Reservoir and fluid properties used in reservoir model. Values are given for reservoir conditions.

Comment [b6]: Were all these valu provided by BP/ Might want to note t in the Table caption or in a note here i addition to the text.

Reservoir or Fluid Property	Value Used in Reservoir Model
Original oil in place	1.1×10^{-8} stock tank barrels
Formation volume factor, B	2.35
Porosity, ϕ	0.21
Effective formation (or pore) compressibility, c_f	1.2×10^{-5} psi^{-1}
Oil viscosity, μ	0.168 cp
Oil compressibility, c_o	1.46×10^{-5} psi^{-1}
Oil density, ρ_o	35.46 lb/ft^3
Water saturation, S_w	0.1
Water compressibility, c_w	3.0×10^{-6} psi^{-1}

Table 2. Values of model parameters estimated from history matching. See Figure 1 for definition of L, W, x_w, and y_w.

Comment [b7]: Are there confidence intervals for these values?

Model Parameter	Estimated Value from History Matching
Horizontal length of reservoir, L	22,270 ft
Horizontal width of reservoir, W	3,830 ft
X-coordinate of Macondo well, x_w	3,100 ft
Y-coordinate of Macondo well, y_w	2,700 ft
permeability, k	570 millidarcy
Coefficient of pressure loss in well, C	3.56×10^6 (barrels/day)^2/psi

Table 3. Simulated values and 95-precent prediction intervals computed by running PEST in predictive analysis mode.

Simulated Quantity	Simulated Value	95% Prediction Interval Min	Max
Initial oil flow rate (stock tank barrels/day)	61,300	60,500	61,900
Final oil flow rate (stock tank barrels/day)	51,000	50,300	51,600
Total volume of oil discharged (stock tank barrels)	4.76×10^6	4.69×10^6	4.81×10^6

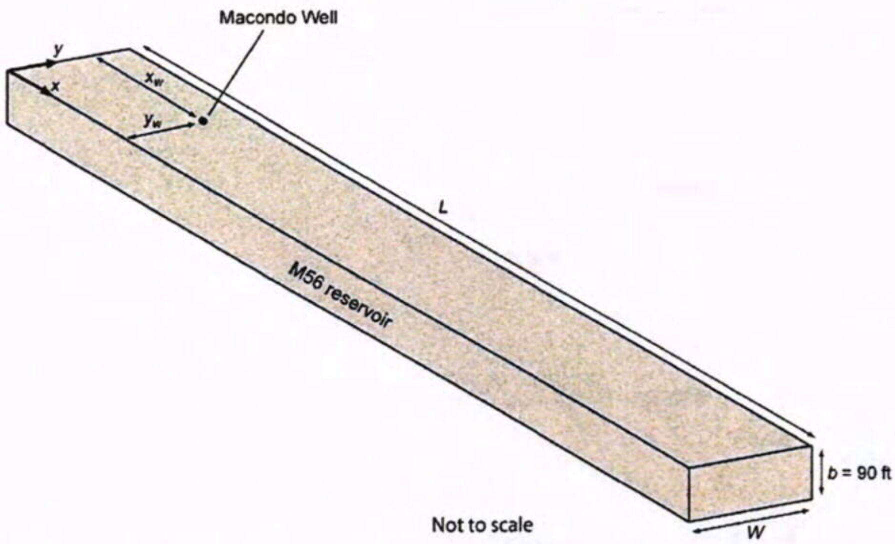

Figure 1. Oblique view of the M56 reservoir.

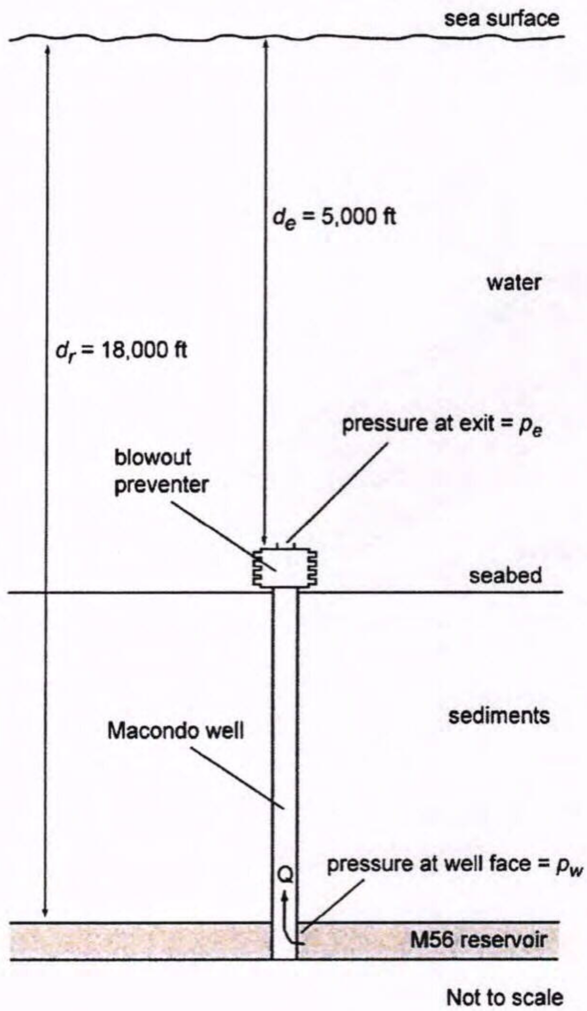

Figure 2. Schematic vertical section showing flow of oil from M56 reservoir through the Macondo well and exiting at the top of the blowout preventer.

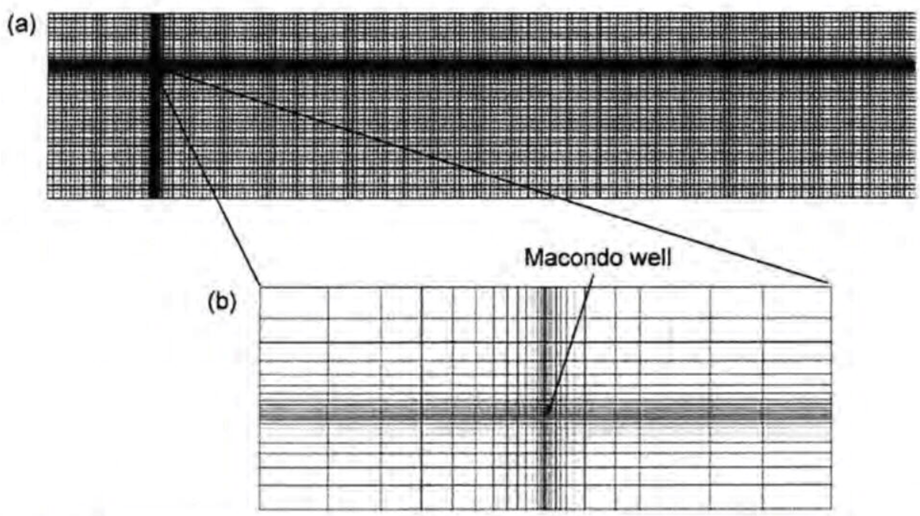

Figure 3. Map view of an example finite-difference grid of the oil reservoir. (a) Entire grid. (b) Detailed view of a small portion of the grid in the vicinity of the Macondo well.

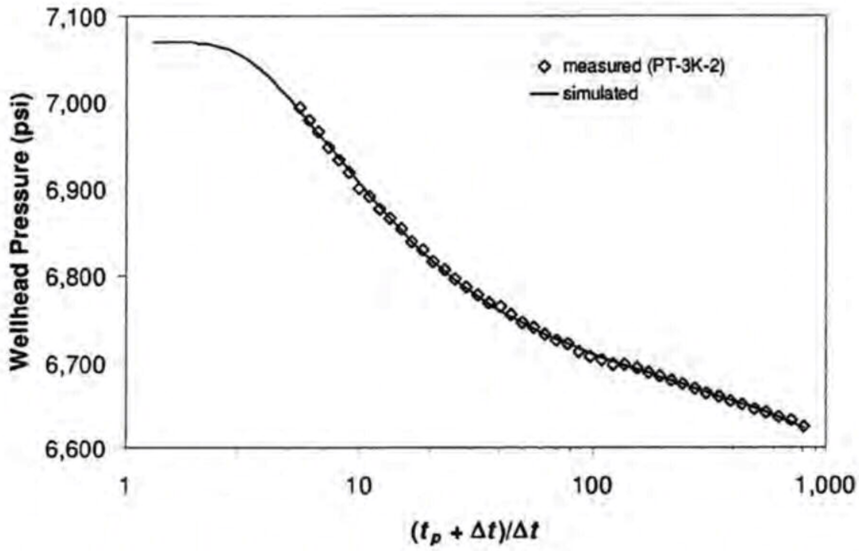

Figure 4. Horner plot of simulated and measured wellhead pressure during Well Integrity Test. t_p is the period of oil flow, which is 86 days. Δt is time since shut in. Note that time increases to the left on the horizontal axis.

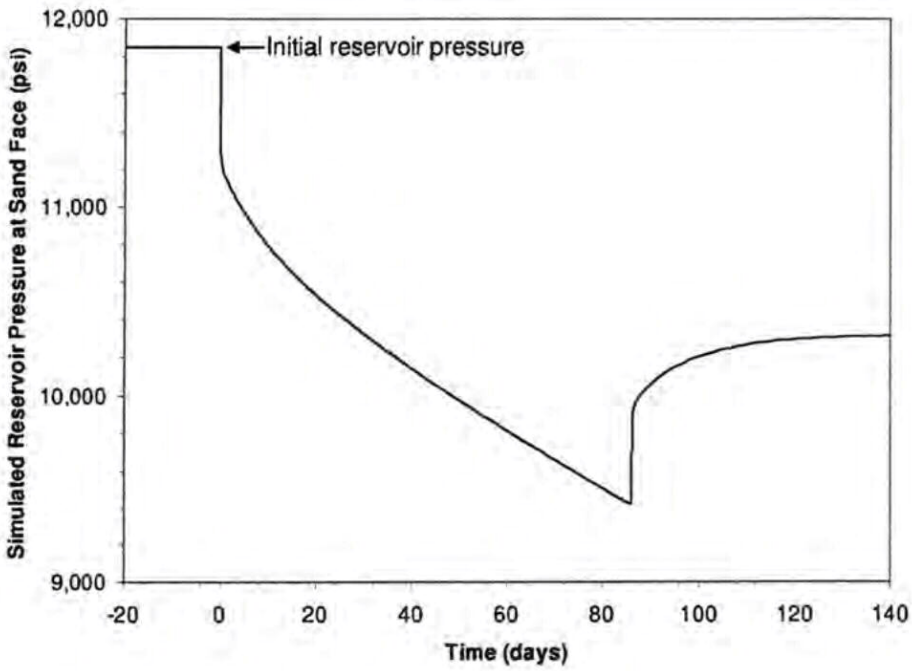

Figure 5. Simulated reservoir pressure at the well face. The origin of the time axis (t = 0) corresponds to April 20, 2010, the date of the Deepwater Horizon blowout.

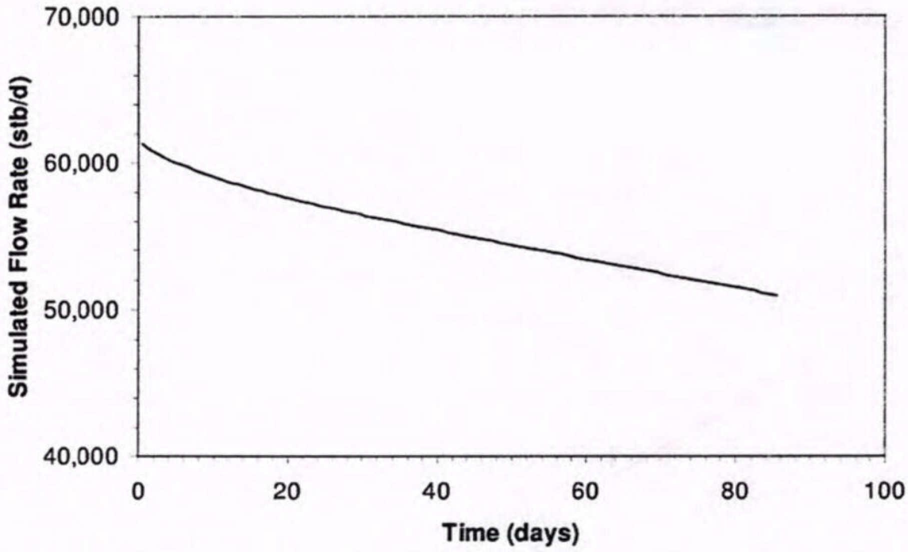

Figure 6. Simulated volumetric flow rate of oil in stock tank barrels per day. The origin of the time axis (t = 0) corresponds to April 20, 2010, the date of the Deepwater Horizon blowout.

Comment [b8]: Consider clarifying that the simulated Q has been converted to volume at the surface by dividing by B